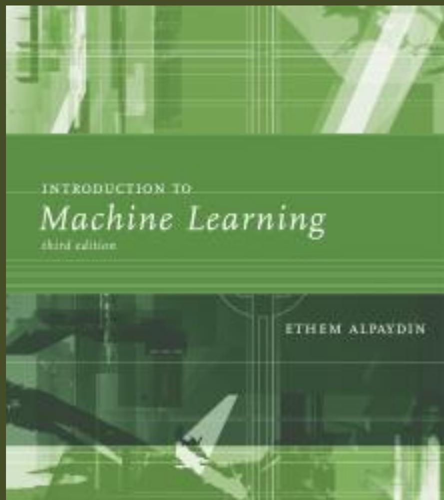

Lecture Slides for

INTRODUCTION TO MACHINE LEARNING

3RD EDITION

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CHAPTER 17:

COMBINING MULTIPLE LEARNERS

Rationale

- No Free Lunch Theorem: There is no algorithm that is always the most accurate
- Generate a group of base-learners which when combined has higher accuracy
- Different learners use different
  - Algorithms
  - Hyperparameters
  - Representations /Modalities/Views
  - Training sets
  - Subproblems
- Diversity vs accuracy

Voting

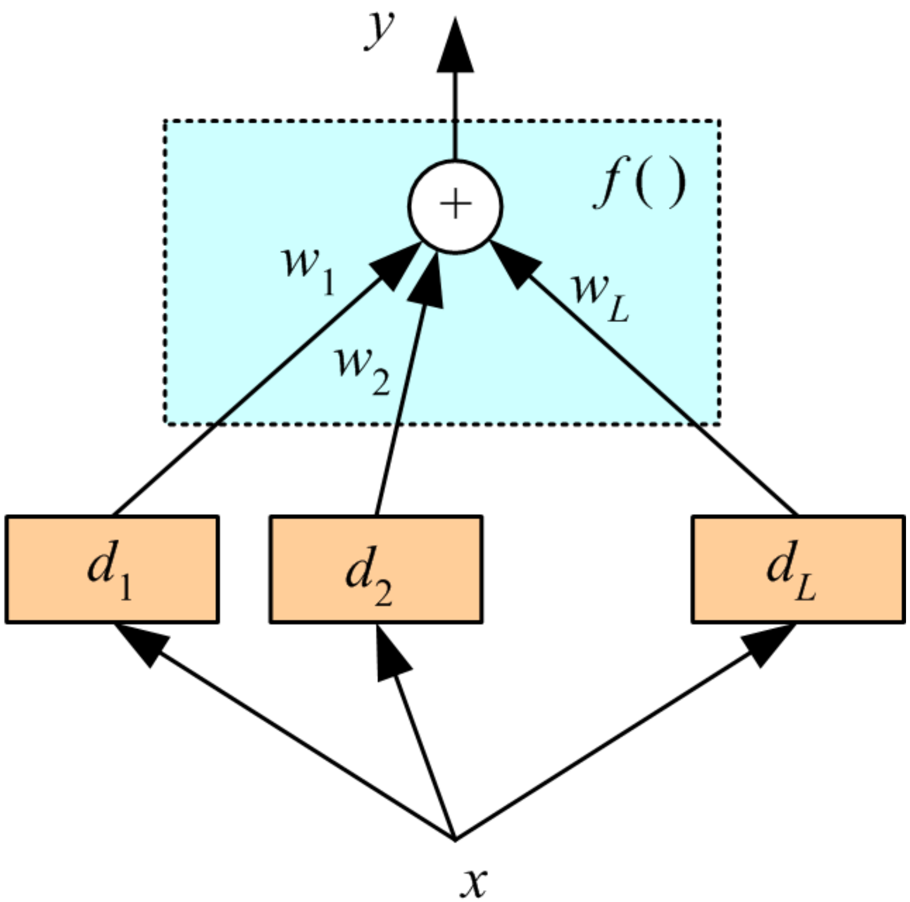
- Linear combination

$$y = \sum_{j=1}^{L} w_j d_j$$

$$w_j \geq 0 \text{ and } \sum_{j=1}^{L} w_j = 1$$

- Classification

$$y_i = \sum_{j=1}^{L} w_j d_{ji}$$



- Bayesian perspective:

$$P(C_i \mid x) = \sum_{\text{all models } \mathcal{M}_j} P(C_i \mid x, \mathcal{M}_j) P(\mathcal{M}_j)$$

- If d_j are iid

$$E[y] = E\left[\sum_j \frac{1}{L} d_j\right] = \frac{1}{L} L \cdot E[d_j] = E[d_j]$$

$$\mathrm{Var}(y) = \mathrm{Var}\left(\sum_j \frac{1}{L} d_j\right) = \frac{1}{L^2} \mathrm{Var}\left(\sum_j d_j\right) = \frac{1}{L^2} L \cdot \mathrm{Var}(d_j) = \frac{1}{L} \mathrm{Var}(d_j)$$

Bias does not change, variance decreases by L

- If dependent, error increase with positive correlation

$$\mathrm{Var}(y) = \frac{1}{L^2} \mathrm{Var}\left(\sum_j d_j\right) = \frac{1}{L^2}\left[\sum_j \mathrm{Var}(d_j) + 2\sum_j \sum_{i<j} Cov(d_i, d_j)\right]$$

Fixed Combination Rules

Rule	Fusion function $f(\cdot)$
Sum	$y_i = \frac{1}{L}\sum_{j=1}^{L} d_{ji}$
Weighted sum	$y_i = \sum_j w_j d_{ji}, w_j \geq 0, \sum_j w_j = 1$
Median	$y_i = \text{median}_j d_{ji}$
Minimum	$y_i = \min_j d_{ji}$
Maximum	$y_i = \max_j d_{ji}$
Product	$y_i = \prod_j d_{ji}$

	C_1	C_2	C_3
d_1	0.2	0.5	0.3
d_2	0.0	0.6	0.4
d_3	0.4	0.4	0.2
Sum	0.2	**0.5**	0.3
Median	0.2	**0.5**	0.4
Minimum	0.0	**0.4**	0.2
Maximum	0.4	**0.6**	0.4
Product	0.0	**0.12**	0.032

Error-Correcting Output Codes

- *K* classes; *L* problems (Dietterich and Bakiri, 1995)
- Code matrix **W** codes classes in terms of learners
- One per class
  L=*K*

$$\mathbf{W} = \begin{bmatrix} +1 & -1 & -1 & -1 \\ -1 & +1 & -1 & -1 \\ -1 & -1 & +1 & -1 \\ -1 & -1 & -1 & +1 \end{bmatrix}$$

- Pairwise
  L=*K*(*K*-1)/2

$$\mathbf{W} = \begin{bmatrix} +1 & +1 & +1 & 0 & 0 & 0 \\ -1 & 0 & 0 & +1 & +1 & 0 \\ 0 & -1 & 0 & -1 & 0 & +1 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix}$$

- Full code $L=2^{(K-1)}-1$

$$\mathbf{W}=\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & +1 & +1 & +1 & +1 \\ -1 & +1 & +1 & -1 & -1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 \end{bmatrix}$$

- With reasonable L, find **W** such that the Hamming distance btw rows and columns are maximized.
- Voting scheme

$$y_i=\sum_{j=1}^{L} w_j d_{ji}$$

- Subproblems may be more difficult than one-per-K

Bagging

- Use bootstrapping to generate *L* training sets and train one base-learner with each (Breiman, 1996)
- Use voting (Average or median with regression)
- Unstable algorithms profit from bagging

AdaBoost

Generate a sequence of base-learners each focusing on previous one's errors

(Freund and Schapire, 1996)

Training:

For all $\{x^t, r^t\}_{t=1}^N \in \mathcal{X}$, initialize $p_1^t = 1/N$

For all base-learners $j = 1, \ldots, L$

Randomly draw $\mathcal{X}_j$ from $\mathcal{X}$ with probabilities p_j^t

Train d_j using $\mathcal{X}_j$

For each (x^t, r^t), calculate $y_j^t \leftarrow d_j(x^t)$

Calculate error rate: $\epsilon_j \leftarrow \sum_t p_j^t \cdot 1(y_j^t \neq r^t)$

If $\epsilon_j > 1/2$, then $L \leftarrow j - 1$; stop

$\beta_j \leftarrow \epsilon_j / (1 - \epsilon_j)$

For each (x^t, r^t), decrease probabilities if correct:

If $y_j^t = r^t$ $p_{j+1}^t \leftarrow \beta_j p_j^t$ Else $p_{j+1}^t \leftarrow p_j^t$

Normalize probabilities:

$Z_j \leftarrow \sum_t p_{j+1}^t; \quad p_{j+1}^t \leftarrow p_{j+1}^t / Z_j$

Testing:

Given x, calculate $d_j(x), j = 1, \ldots, L$

Calculate class outputs, $i = 1, \ldots, K$:

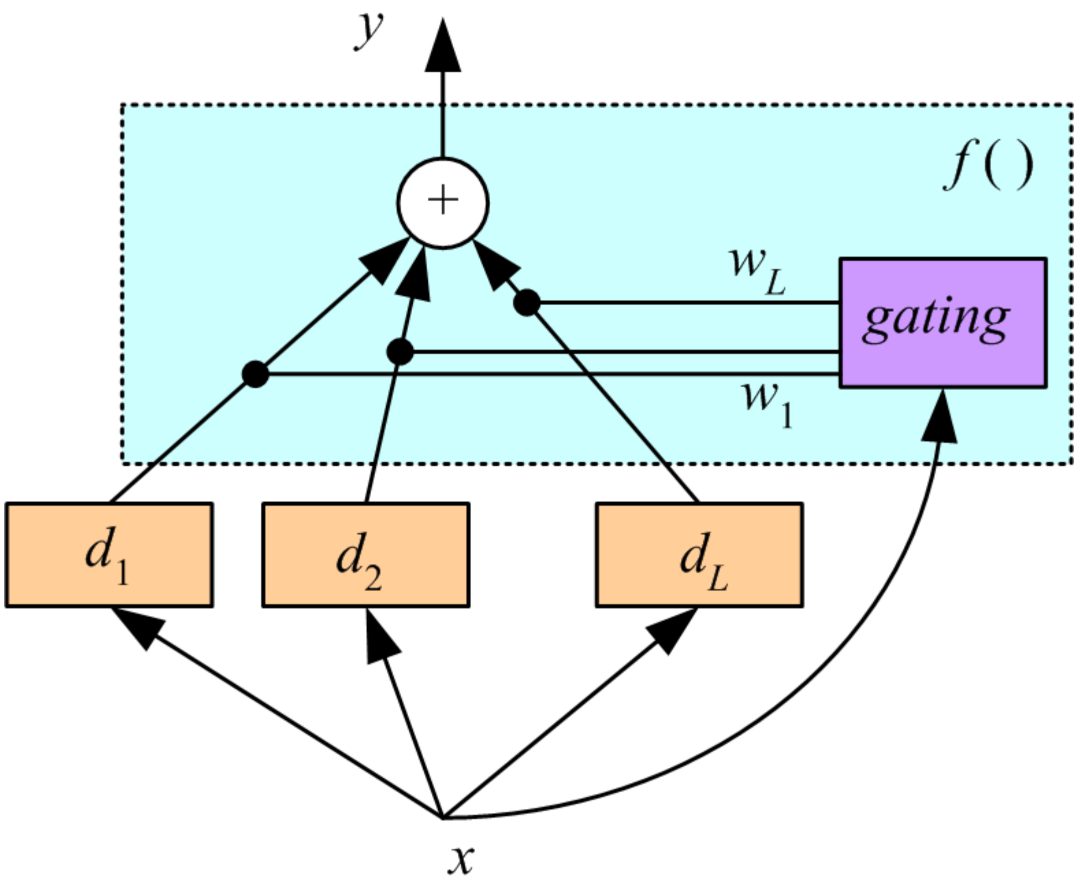
$$y_i = \sum_{j=1}^L \left(\log \frac{1}{\beta_j}\right) d_{ji}(x)$$

Mixture of Experts

Voting where weights are input-dependent (gating)

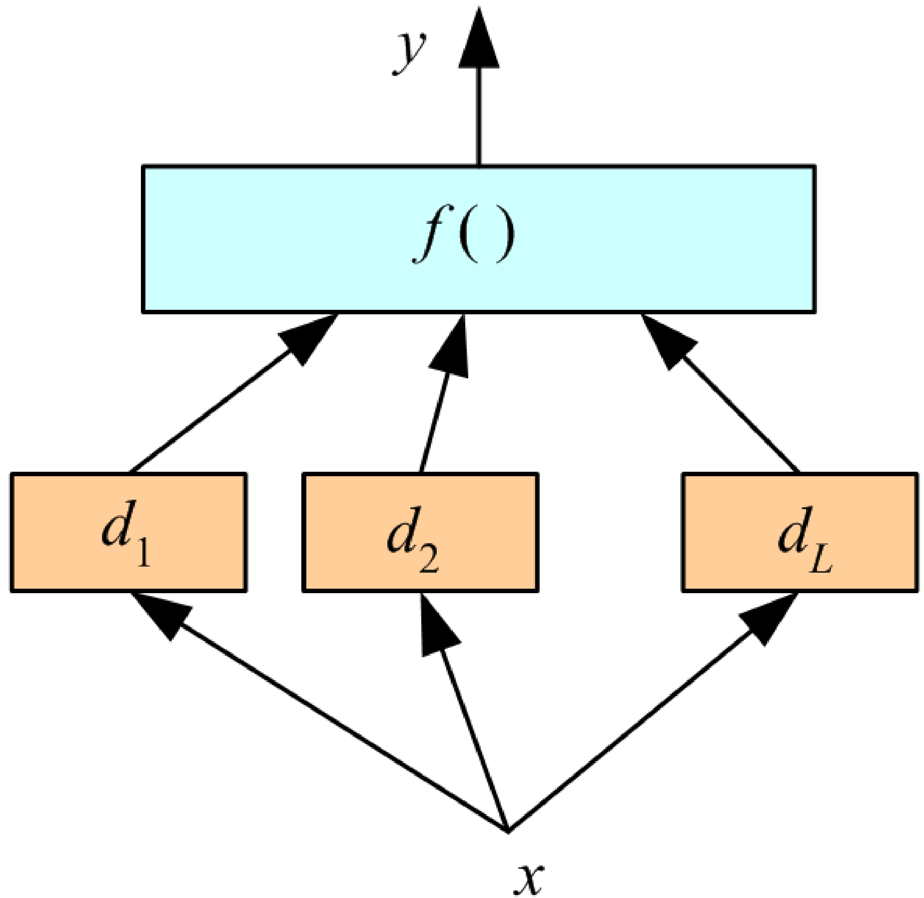
$$y = \sum_{j=1}^{L} w_j d_j$$

(Jacobs et al., 1991)
Experts or gating
can be nonlinear



Stacking

- Combiner f () is another learner (Wolpert, 1992)



Fine-Tuning an Ensemble

- Given an ensemble of dependent classifiers, do not use it as is, try to get independence

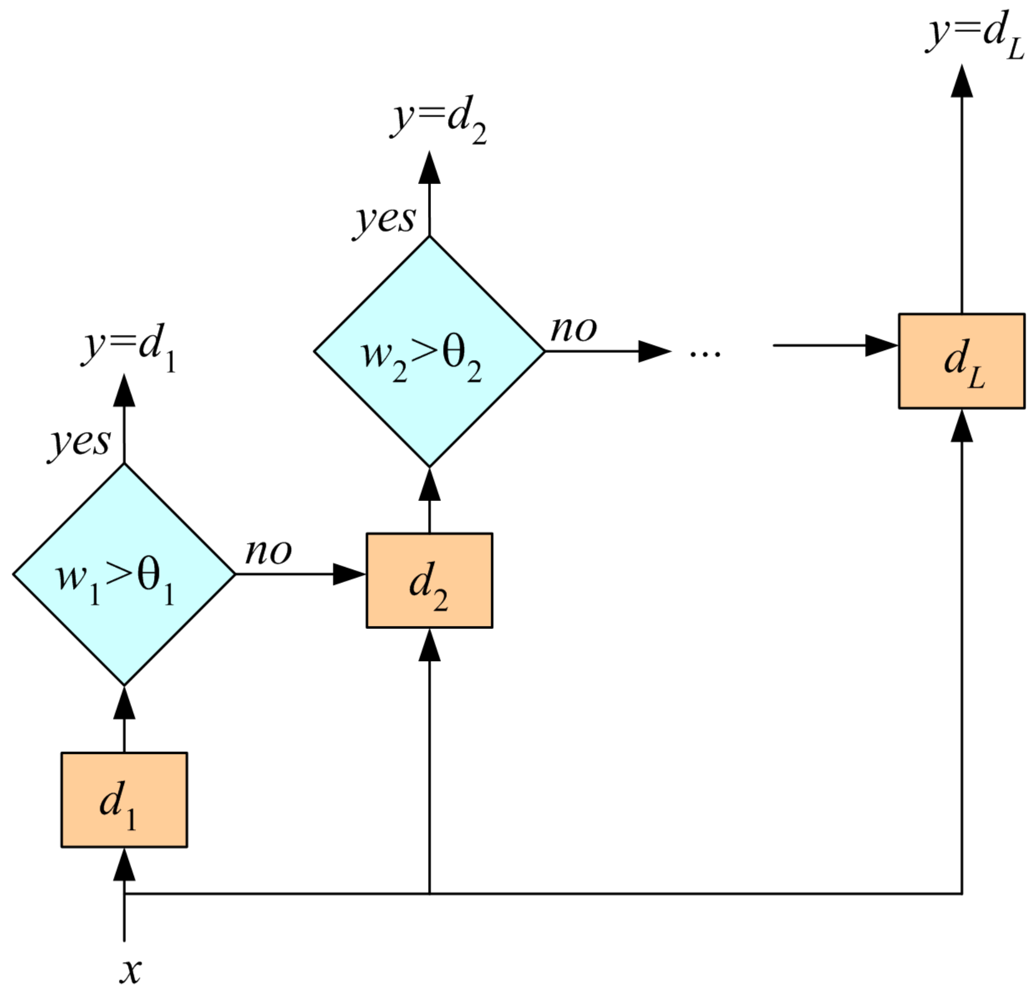
1. Subset selection: Forward (growing)/Backward (pruning) approaches to improve accuracy/diversity/independence
2. Train metaclassifiers: From the output of correlated classifiers, extract new combinations that are uncorrelated. Using PCA, we get “eigenlearners.”

- Similar to feature selection vs feature extraction

Cascading

Use d_j only if preceding ones are not confident

Cascade learners in order of complexity



Combining Multiple Sources/Views

- Early integration: Concat all features and train a single learner
- Late integration: With each feature set, train one learner, then either use a fixed rule or stacking to combine decisions
- Intermediate integration: With each feature set, calculate a kernel, then use a single SVM with multiple kernels
- Combining features vs decisions vs kernels