Introduction to Machine Learning

Figures

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Chapter 1:
Introduction
Figure 1.1: Example of a training dataset where each circle corresponds to one data instance with input values in the corresponding axes and its sign indicates the class. For simplicity, only two customer attributes, income and savings, are taken as input and the two classes are low-risk (‘+’) and high-risk (‘−’). An example discriminant that separates the two types of examples is also shown. *From:* E. Alpaydın. 2004. *Introduction to Machine Learning.* © *The MIT Press.*
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Chapter 2:

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Chapter 3:

Bayes Decision Theory
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Chapter 4:
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Figure 4.6: In the same setting as that of figure 4.5, using one hundred models instead of five, bias, variance, and error for polynomials of order 1 to 5. Order 1 has the smallest variance. Order 5 has the smallest bias. As the order is increased, bias decreases but variance increases. Order 3 has the minimum error. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 4.7: In the same setting as that of figure 4.5, training and validation sets (each containing 50 instances) are generated. (a) Training data and fitted polynomials of order from 1 to 8. (b) Training and validation errors as a function of the polynomial order. The “elbow” is at 3. From: E. Alpaydin. 2004. Introduction to Machine Learning. © The MIT Press.
Chapter 5:

Multivariate Methods
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Figure 5.6: All classes have equal, diagonal covariance matrices of equal variances on both dimensions. *From: E. Alpaydın. 2004. Introduction to Machine Learning.* ©The MIT Press.
Chapter 6:

Dimensionality Reduction
Figure 6.1: Principal components analysis centers the sample and then rotates the axes to line up with the directions of highest variance. If the variance on $z_2$ is too small, it can be ignored and we have dimensionality reduction from two to one. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 6.2: (a) Scree graph. (b) Proportion of variance explained is given for the Optdigits dataset from the UCI Repository. This is a handwritten digit dataset with ten classes and sixty-four dimensional inputs. The first twenty eigenvectors explain 90 percent of the variance. *From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.*
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Figure 6.4: Principal components analysis generates new variables that are linear combinations of the original input variables. In factor analysis, however, we posit that there are factors that when linearly combined generate the input variables. *From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.*
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Figure 6.7: Two-dimensional, two-class data projected on $\mathbf{w}$. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 6.8: Optdigits data plotted in the space of the first two dimensions found by LDA. Comparing this with figure 6.3, we see that LDA, as expected, leads to a better separation of classes than PCA. Even in this two-dimensional space (there are nine), we can discern separate clouds for different classes.

Chapter 7: Clustering
Figure 7.1: Given $x$, the encoder sends the index of the closest code word and the decoder generates the code word with the received index as $x'$. Error is $\|x' - x\|^2$. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Initialize $m_i, i = 1, \ldots, k$, for example, to $k$ random $x^t$

Repeat

For all $x^t \in \mathcal{X}$

$$b^t_i \left\{ \begin{array}{ll} 1 & \text{if } \|x^t - m_i\| = \min_j \|x^t - m_j\| \\ 0 & \text{otherwise} \end{array} \right.$$ 

For all $m_i, i = 1, \ldots, k$

$$m_i \leftarrow \frac{\sum_t b^t_i x^t}{\sum_t b^t_i}$$

Until $m_i$ converge

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Figure 7.4: Data points and the fitted Gaussians by EM, initialized by one $k$-means iteration of figure 7.2. Unlike in $k$-means, as can be seen, EM allows estimating the covariance matrices. The data points labeled by greater $h_i$, the contours of the estimated Gaussian densities, and the separating curve of $h_i = 0.5$ (dashed line) are shown. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 7.5: A two-dimensional dataset and the dendrogram showing the result of single-link clustering is shown. Note that leaves of the tree are ordered so that no branches cross. The tree is then intersected at a desired value of $h$ to get the clusters.

Chapter 8:

Nonparametric Methods
Figure 8.2: Naive estimate for various bin lengths.

Figure 8.3: Kernel estimate for various bin lengths. 
Figure 8.4: $k$-nearest neighbor estimate for various $k$ values. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 8.5: Dotted lines are the Voronoi tessellation and the straight line is the class discriminant. In condensed nearest neighbor, those instances that do not participate in defining the discriminant (marked by ‘*’) can be removed without increasing the training error. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
\[ Z \leftarrow \emptyset \]
Repeat
\[ \text{For all } x \in X \text{ (in random order)} \]
\[ \text{Find } x' \in Z \text{ s.t. } \|x - x'\| = \min_{x_j \in Z} \|x - x_j\| \]
\[ \text{If class}(x) \neq \text{class}(x') \text{ add } x \text{ to } Z \]
Until \( Z \) does not change

Figure 8.6: Condensed nearest neighbor algorithm.

Figure 8.7: Regressograms for various bin lengths. ‘×’ denote data points. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 8.9: Kernel smooth for various bin lengths.

Figure 8.11: Regressograms with linear fits in bins for various bin lengths. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Chapter 9:
Decision Trees
Figure 9.1: Example of a dataset and the corresponding decision tree. Oval nodes are the decision nodes and rectangles are leaf nodes. The univariate decision node splits along one axis, and successive splits are orthogonal to each other. After the first split, $\{x_1 < w_{10}\}$ is pure and is not split further. From: E. Alpaydîn. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 9.2: Entropy function for a two-class problem.

GenerateTree(\mathcal{X})
    If NodeEntropy(\mathcal{X}) < \theta_I /* eq. 9.3
        Create leaf labelled by majority class in \mathcal{X}
        Return
    i ← SplitAttribute(\mathcal{X})
    For each branch of \mathbf{x}_i
        Find \mathcal{X}_i falling in branch
        GenerateTree(\mathcal{X}_i)

SplitAttribute(\mathcal{X})
    MinEnt ← MAX
    For all attributes \( i = 1, \ldots, d \)
        If \mathbf{x}_i is discrete with \( n \) values
            Split \mathcal{X} into \( \mathcal{X}_1, \ldots, \mathcal{X}_n \) by \mathbf{x}_i
            \( e \leftarrow \text{SplitEntropy}(\mathcal{X}_1, \ldots, \mathcal{X}_n) /* eq. 9.8 */ \)
            If \( e < \text{MinEnt} \) \( \text{MinEnt} \leftarrow e; \text{bestf} \leftarrow i \)
        Else /* \mathbf{x}_i is numeric */
            For all possible splits
                Split \mathcal{X} into \( \mathcal{X}_1, \mathcal{X}_2 \) on \mathbf{x}_i
                \( e \leftarrow \text{SplitEntropy}(\mathcal{X}_1, \mathcal{X}_2) \)
                If \( e < \text{MinEnt} \) \( \text{MinEnt} \leftarrow e; \text{bestf} \leftarrow i \)
    Return \text{bestf}

Figure 9.3: Classification tree construction. From:
Figure 9.4: Regression tree smooths for various values of $\theta_r$. The corresponding trees are given in figure 9.5. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 9.5: Regression trees implementing the smooths of figure 9.4 for various values of $\theta_r$. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 9.6: Example of a (hypothetical) decision tree. Each path from the root to a leaf can be written down as a conjunctive rule, composed of conditions defined by the decision nodes on the path.

Ripper(Pos,Neg,k)
    RuleSet ← LearnRuleSet(Pos,Neg)
    For k times
        RuleSet ← OptimizeRuleSet(RuleSet,Pos,Neg)
    LearnRuleSet(Pos,Neg)
    RuleSet ← ∅
    DL ← DescLen(RuleSet,Pos,Neg)
    Repeat
        Rule ← LearnRule(Pos,Neg)
        Add Rule to RuleSet
        DL' ← DescLen(RuleSet,Pos,Neg)
        If DL'>DL+64
            PruneRuleSet(RuleSet,Pos,Neg)
            Return RuleSet
        If DL'<DL DL ← DL'
            Delete instances covered from Pos and Neg
    Until Pos = ∅
    Return RuleSet

Figure 9.7: Ripper algorithm for learning rules.

PruneRuleSet(RuleSet,Pos,Neg)
    For each Rule ∈ RuleSet in reverse order
    DL ← DescLen(RuleSet,Pos,Neg)
    DL’ ← DescLen(RuleSet-Rule,Pos,Neg)
    IF DL’¡DL Delete Rule from RuleSet
    Return RuleSet
OptimizeRuleSet(RuleSet,Pos,Neg)
    For each Rule ∈ RuleSet
        DL0 ← DescLen(RuleSet,Pos,Neg)
        DL1 ← DescLen(RuleSet-Rule+ReplaceRule(RuleSet,Pos,Neg),Pos,Neg)
        DL2 ← DescLen(RuleSet-Rule+ReviseRule(RuleSet,Rule,Pos,Neg),Pos,Neg)
        If DL1=min(DL0,DL1,DL2)
            Delete Rule from RuleSet and add ReplaceRule(RuleSet,Pos,Neg)
        Else If DL2=min(DL0,DL1,DL2)
            Delete Rule from RuleSet and add ReviseRule(RuleSet,Rule,Pos,Neg)
    Return RuleSet

Figure 9.8: Example of a linear multivariate decision tree. The linear multivariate node can place an arbitrary hyperplane thus is more general, whereas the univariate node is restricted to axis-aligned splits. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Chapter 10:
Linear Discrimination
Figure 10.1: In the two-dimensional case, the linear discriminant is a line that separates the examples from two classes. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 10.3: In linear classification, each hyperplane $H_i$ separates the examples of $C_i$ from the examples of all other classes. Thus for it to work, the classes should be linearly separable. Dotted lines are the induced boundaries of the linear classifier. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 10.4: In pairwise linear separation, there is a separate hyperplane for each pair of classes. For an input to be assigned to $C_1$, it should be on the positive side of $H_{12}$ and $H_{13}$ (which is the negative side of $H_{31}$); we do not care for the value of $H_{23}$. In this case, $C_1$ is not linearly separable from other classes but is pairwise linearly separable. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 10.5: The logistic, or sigmoid, function.

For $j = 0, \ldots, d$

\[ w_j \leftarrow \text{rand}(-0.01, 0.01) \]

Repeat

For $j = 0, \ldots, d$

\[ \Delta w_j \leftarrow 0 \]

For $t = 1, \ldots, N$

\[ o \leftarrow 0 \]

For $j = 0, \ldots, d$

\[ o \leftarrow o + w_j x^t_j \]

\[ y \leftarrow \text{sigmoid}(o) \]

\[ \Delta w_j \leftarrow \Delta w_j + (r^t - y)x^t_j \]

For $j = 0, \ldots, d$

\[ w_j \leftarrow w_j + \eta \Delta w_j \]

Until convergence

Figure 10.6: Logistic discrimination algorithm implementing gradient-descent for the single output case with two classes. For $w_0$, we assume that there is an extra input $x_0$, which is always $+1$: $x^t_0 \equiv +1, \forall t$.  

Figure 10.7: For a univariate two-class problem (shown with ‘◦’ and ‘×’), the evolution of the line $wx + w_0$ and the sigmoid output after 10, 100, and 1,000 iterations over the sample. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
For $i = 1, \ldots, K$, For $j = 0, \ldots, d$, $w_{ij} \leftarrow \text{rand}(-0.01, 0.01)$

Repeat

For $i = 1, \ldots, K$, For $j = 0, \ldots, d$, $\Delta w_{ij} \leftarrow 0$

For $t = 1, \ldots, N$

For $i = 1, \ldots, K$

\[ o_i \leftarrow 0 \]

For $j = 0, \ldots, d$

\[ o_i \leftarrow o_i + w_{ij}x_t^j \]

For $i = 1, \ldots, K$

\[ y_i \leftarrow \exp(o_i)/\sum_k \exp(o_k) \]

For $i = 1, \ldots, K$

For $j = 0, \ldots, d$

\[ \Delta w_{ij} \leftarrow \Delta w_{ij} + (r_t^i - y_i)x_t^j \]

For $i = 1, \ldots, K$

For $j = 0, \ldots, d$

\[ w_{ij} \leftarrow w_{ij} + \eta \Delta w_{ij} \]

Until convergence

Figure 10.8: Logistic discrimination algorithm implementing gradient-descent for the case with $K > 2$ classes. For generality, we take $x_0^t \equiv 1, \forall t$.

Figure 10.9: For a two-dimensional problem with three classes, the solution found by logistic discrimination. Thin lines are where $g_i(x) = 0$, and the thick line is the boundary induced by the linear classifier choosing the maximum. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 10.10: For the same example in figure 10.9, the linear discriminants (top), and the posterior probabilities after the softmax (bottom). From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 10.11: On both sides of the optimal separating hyperplane, the instances are at least \( \frac{1}{\|w\|} \) away and the total margin is \( \frac{2}{\|w\|} \). From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 10.12: In classifying an instance, there are three possible cases: In (1), $\xi = 0$; it is on the right side and sufficiently away. In (2), $\xi = 1 + g(x) > 1$; it is on the wrong side. In (3), $\xi = 1 - g(x), 0 < \xi < 1$; it is on the right side but is in the margin and not sufficiently away. From: E. Alpaydîn. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 10.13: Quadratic and $\epsilon$-sensitive error functions. We see that $\epsilon$-sensitive error function is not affected by small errors and also is affected less by large errors and thus is more robust to outliers. 

Chapter 11:

Multilayer Perceptrons
Figure 11.1: Simple perceptron. $x_j, j = 1, \ldots, d$ are the input units. $x_0$ is the bias unit that always has the value 1. $y$ is the output unit. $w_j$ is the weight of the directed connection from input $x_j$ to the output.

Figure 11.2: $K$ parallel perceptrons. $x_j, j = 0, \ldots, d$ are the inputs and $y_i, i = 1, \ldots, K$ are the outputs. $w_{ij}$ is the weight of the connection from input $x_j$ to output $y_i$. Each output is a weighted sum of the inputs. When used for $K$-class classification problem, there is a postprocessing to choose the maximum, or softmax if we need the posterior probabilities. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
For $i = 1, \ldots, K$
   For $j = 0, \ldots, d$
      $w_{ij} \leftarrow \text{rand}(-0.01, 0.01)$
Repeat
   For all $(\mathbf{x}^t, r^t) \in \mathcal{X}$ in random order
      For $i = 1, \ldots, K$
         $o_i \leftarrow 0$
      For $j = 0, \ldots, d$
         $o_i \leftarrow o_i + w_{ij} x_j^t$
   For $i = 1, \ldots, K$
      $y_i \leftarrow \exp(o_i) / \sum_k \exp(o_k)$
   For $i = 1, \ldots, K$
      For $j = 0, \ldots, d$
         $w_{ij} \leftarrow w_{ij} + \eta (r_i^t - y_i) x_j^t$
Until convergence

Figure 11.3: Percepton training algorithm implementing stochastic online gradient-descent for the case with $K > 2$ classes. This is the online version of the algorithm given in figure 10.8. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 11.5: XOR problem is not linearly separable. We cannot draw a line where the empty circles are on one side and the filled circles on the other side.

Figure 11.7: The multilayer perceptron that solves the XOR problem. The hidden units and the output have the threshold activation function with threshold at 0. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 11.8: Sample training data shown as ‘+’, where $x^t \sim U(-0.5, 0.5)$, and $y^t = f(x^t) + \mathcal{N}(0, 0.1)$. $f(x) = \sin(6x)$ is shown by a dashed line. The evolution of the fit of an MLP with two hidden units after 100, 200, and 300 epochs is drawn. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 11.9: The mean square error on training and validation sets as a function of training epochs.  

Figure 11.10: (a) The hyperplanes of the hidden unit weights on the first layer, (b) hidden unit outputs, and (c) hidden unit outputs multiplied by the weights on the second layer. Two sigmoid hidden units slightly displaced, one multiplied by a negative weight, when added, implement a bump. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Initialize all $v_{ih}$ and $w_{hj}$ to $\text{rand}(-0.01, 0.01)$

Repeat

For all $(x^t, r^t) \in \mathcal{X}$ in random order

For $h = 1, \ldots, H$

\[ z_h \leftarrow \text{sigmoid}(w_h^T x^t) \]

For $i = 1, \ldots, K$

\[ y_i = v_i^T z \]

For $i = 1, \ldots, K$

\[ \Delta v_i = \eta (r_i^t - y_i^t) z \]

For $h = 1, \ldots, H$

\[ \Delta w_h = \eta \left( \sum_i (r_i^t - y_i^t) v_{ih} \right) z_h (1 - z_h) x^t \]

For $i = 1, \ldots, K$

\[ v_i \leftarrow v_i + \Delta v_i \]

For $h = 1, \ldots, H$

\[ w_h \leftarrow w_h + \Delta w_h \]

Until convergence

Figure 11.11: Backpropagation algorithm for training a multilayer perceptron for regression with $K$ outputs. This code can easily be adapted for two-class classification (by setting a single sigmoid output) and to $K > 2$ classification (by using softmax outputs). From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 11.12: As complexity increases, training error is fixed but the validation error starts to increase and the network starts to overfit. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 11.13: As training continues, the validation error starts to increase and the network starts to overfit. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 11.14: A structured MLP. Each unit is connected to a local group of units below it and checks for a particular feature—for example, edge, corner, and so forth—in vision. Only one hidden unit is shown for each region; typically there are many to check for different local features. *From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.*
Figure 11.15: In weight sharing, different units have connections to different inputs but share the same weight value (denoted by line type). Only one set of units is shown; there should be multiple sets of units, each checking for different features. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 11.16: The identity of the object does not change when it is translated, rotated, or scaled. Note that this may not always be true, or may be true up to a point: ‘b’ and ‘q’ are rotated versions of each other. These are hints that can be incorporated into the learning process to make learning easier. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 11.17: Two examples of constructive algorithms: Dynamic node creation adds a unit to an existing layer. Cascade correlation adds each unit as a new hidden layer connected to all the previous layers. Dashed lines denote the newly added unit/connections. Bias units/weights are omitted for clarity. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 11.18: Optdigits data plotted in the space of the two hidden units of an MLP trained for classification. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 11.19: In the autoassociator, there are as many outputs as there are inputs and the desired outputs are the inputs. When the number of hidden units is less than the number of inputs, the MLP is trained to find the best coding of the inputs on the hidden units, performing dimensionality reduction. On the left, the first layer acts as an encoder and the second layer acts as the decoder. On the right, if the encoder and decoder are multilayer perceptrons with sigmoid hidden units, the network performs nonlinear dimensionality reduction. From: E. Alpaydin. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 11.20: A time delay neural network. Inputs in a time window of length $T$ are delayed in time until we can feed all $T$ inputs as the input vector to the MLP. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 11.21: Examples of MLP with partial recurrency. Recurrent connections are shown with dashed lines: (a) self-connections in the hidden layer, (b) self-connections in the output layer, and (c) connections from the output to the hidden layer. Combinations of these are also possible. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 11.22: Backpropagation through time: (a) recurrent network and (b) its equivalent unfolded network that behaves identically in four steps. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Chapter 12:

Local Models
Figure 12.1: Shaded circles are the centers and the empty circle is the input instance. The online version of \( k \)-means moves the closest center along the direction of \((x - m_i)\) by a factor specified by \( \eta \).

Initialize $m_i, i = 1, \ldots, k$, for example, to $k$ random $x^t$

Repeat

For all $x^t \in X$ in random order

\[ i \leftarrow \arg \min_j \| x^t - m_j \| \]

\[ m_i \leftarrow m_i + \eta(x^t - m_j) \]

Until $m_i$ converge

Figure 12.2: Online $k$-means algorithm. The batch version is given in figure 7.3. From: E. Alpaydin. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 12.3: The winner-take-all competitive neural network, which is a network of $k$ perceptrons with recurrent connections at the output. Dashed lines are recurrent connections, of which the ones that end with an arrow are excitatory and the ones that end with a circle are inhibitory. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 12.4: The distance from $x^a$ to the closest center is less than the vigilance value $\rho$ and the center is updated as in online $k$-means. However, $x^b$ is not close enough to any of the centers and a new group should be created at that position. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 12.5: In the SOM, not only the closest unit but also its neighbors, in terms of indices, are moved toward the input. Here, neighborhood is 1; $m_i$ and its 1-nearest neighbors are updated. Note here that $m_{i+1}$ is far from $m_i$, but as it is updated with $m_i$, and as $m_i$ will be updated when $m_{i+1}$ is the winner, they will become neighbors in the input space as well.

Figure 12.6: The one-dimensional form of the bell-shaped function used in the radial basis function network. This one has $m = 0$ and $s = 1$. It is like a Gaussian but it is not a density; it does not integrate to 1. It is nonzero between $(m - 3s, m + 3s)$, but a more conservative interval is $(m - 2s, m + 2s)$. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Local representation in the space of \((p_1, p_2, p_3)\)

\[ x^a : (1.0, 0.0, 0.0) \]
\[ x^b : (0.0, 0.0, 1.0) \]
\[ x^c : (1.0, 1.0, 0.0) \]

Distributed representation in the space of \((h_1, h_2)\)

\[ x^a : (1.0, 1.0) \]
\[ x^b : (0.0, 1.0) \]
\[ x^c : (1.0, 0.0) \]

Figure 12.7: The difference between local and distributed representations. The values are hard, 0/1, values. One can use soft values in \((0, 1)\) and get a more informative encoding. In the local representation, this is done by the Gaussian RBF that uses the distance to the center, \(m_i\), and in the distributed representation, this is done by the sigmoid that uses the distance to the hyperplane, \(w_i\). From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 12.8: The RBF network where $p_h$ are the hidden units using the bell-shaped activation function. $m_h, s_h$ are the first-layer parameters, and $w_i$ are the second-layer weights. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 12.9: (-) Before and (- -) after normalization for three Gaussians whose centers are denoted by ‘*’. Note how the nonzero region of a unit depends also on the positions of other units. If the spreads are small, normalization implements a harder split; with large spreads, units overlap more. From: *E. Alpaydìn*. 2004. Introduction to Machine Learning. *© The MIT Press.*
Figure 12.10: The mixture of experts can be seen as an RBF network where the second-layer weights are outputs of linear models. Only one linear model is shown for clarity. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 12.11: The mixture of experts can be seen as a model for combining multiple models. $w_h$ are the models and the gating network is another model determining the weight of each model, as given by $g_h$. Viewed in this way, neither the experts nor the gating are restricted to be linear. From: E. Alpaydín. 2004. Introduction to Machine Learning. ©The MIT Press.
Chapter 13:
Hidden Markov Models
Figure 13.1: Example of a Markov model with three states is a stochastic automaton. $\pi_i$ is the probability that the system starts in state $S_i$, and $a_{ij}$ is the probability that the system moves from state $S_i$ to state $S_j$. From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 13.2: An HMM unfolded in time as a lattice (or trellis) showing all the possible trajectories. One path, shown in thicker lines, is the actual (unknown) state trajectory that generated the observation sequence. From: E. Alpaydin. 2004. Introduction to Machine Learning. ©The MIT Press.
Figure 13.3: Forward-backward procedure: (a) computation of $\alpha_t(j)$ and (b) computation of $\beta_t(i)$.

Figure 13.4: Computation of arc probabilities, $\xi_t(i, j)$.

Figure 13.5: Example of a left-to-right HMM. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Chapter 14:
Assessing and Comparing
Classification Algorithms
Figure 14.1: Typical roc curve. Each classifier has a parameter, for example, a threshold, which allows us to move over this curve, and we decide on a point, based on the relative importance of hits versus false alarms, namely, true positives and false positives.

Figure 14.2: 95 percent of the unit normal distribution lies between $-1.96$ and $1.96$. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 14.3: 95 percent of the unit normal distribution lies before 1.64. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Chapter 15:

Combining Multiple Learners
Figure 15.1: In voting, the combiner function $f(\cdot)$ is a weighted sum. $d_j$ are the multiple learners, and $w_j$ are the weights of their votes. $y$ is the overall output. In the case of multiple outputs, for example, classification, the learners have multiple outputs $d_{ji}$ whose weighted sum gives $y_i$. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Training:
For all \( \{x^t, r^t\}_{t=1}^N \in \mathcal{X} \), initialize \( p_1^t = 1/N \)
For all base-learners \( j = 1, \ldots, L \)
Randomly draw \( \mathcal{X}_j \) from \( \mathcal{X} \) with probabilities \( p_j^t \)
Train \( d_j \) using \( \mathcal{X}_j \)
For each \( (x^t, r^t) \), calculate \( y_j^t \leftarrow d_j(x^t) \)
Calculate error rate: \( \epsilon_j \leftarrow \sum_t p_j^t \cdot 1(y_j^t \neq r^t) \)
If \( \epsilon_j > 1/2 \), then \( L \leftarrow j - 1 \); stop
\( \beta_j \leftarrow \epsilon_j / (1 - \epsilon_j) \)
For each \( (x^t, r^t) \), decrease probabilities if correct:
If \( y_j^t = r^t \) \( p_{j+1}^t \leftarrow \beta_j p_j^t \) Else \( p_{j+1}^t \leftarrow p_j^t \)
Normalize probabilities:
\( Z_j \leftarrow \sum_t p_{j+1}^t; \ p_{j+1}^t \leftarrow p_{j+1}^t / Z_j \)
Testing:
Given \( x \), calculate \( d_j(x), j = 1, \ldots, L \)
Calculate class outputs, \( i = 1, \ldots, K \):
\[
y_i = \sum_{j=1}^L \left( \log \frac{1}{\beta_j} \right) d_{ji}(x)
\]

Figure 15.2: AdaBoost algorithm. From:
Figure 15.3: Mixture of experts is a voting method where the votes, as given by the gating system, are a function of the input. The combiner system $f$ also includes this gating system. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 15.4: In stacked generalization, the combiner is another learner and is not restricted to being a linear combination as in voting. *From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.*
Figure 15.5: Cascading is a multistage method where there is a sequence of classifiers, and the next one is used only when the preceding ones are not confident. 

Chapter 16:

Reinforcement Learning
Figure 16.1: The agent interacts with an environment. At any state of the environment, the agent takes an action that changes the state and returns a reward. *From: E. Alpaydın. 2004. Introduction to Machine Learning. ©The MIT Press.*
Initialize $V(s)$ to arbitrary values
Repeat
  For all $s \in S$
    For all $a \in A$
      $Q(s, a) \leftarrow E[r|s, a] + \gamma \sum_{s' \in S} P(s'|s, a)V(s')$
      $V(s) \leftarrow \max_a Q(s, a)$
  Until $V(s)$ converge

Initialize a policy $\pi$ arbitrarily

Repeat

$\pi \leftarrow \pi'$

Compute the values using $\pi$ by solving the linear equations

$$V^\pi(s) = E[r|s, \pi(s)] + \gamma \sum_{s' \in S} P(s'|s, \pi(s))V^\pi(s')$$

Improve the policy at each state

$$\pi'(s) \leftarrow \arg \max_a (E[r|s, a] + \gamma \sum_{s' \in S} P(s'|s, a)V^\pi(s'))$$

Until $\pi = \pi'$

---

Figure 16.4: Example to show that $Q$ values increase but never decrease. This is a deterministic grid-world where $G$ is the goal state with reward 100, all other immediate rewards are 0 and $\gamma = 0.9$. Let us consider the $Q$ value of the transition marked by asterisk, and let us just consider only the two paths $A$ and $B$. Let us say that path $A$ is seen before path $B$, then we have $\gamma \max(0, 81) = 72.9$. If afterward $B$ is seen, a shorter path is found and the $Q$ value becomes $\gamma \max(100, 81) = 90$. If $B$ is seen before $A$, the $Q$ value is $\gamma \max(100, 0) = 90$. Then when $B$ is seen, it does not change because $\gamma \max(100, 81) = 90$. From: E. Alpaydin. 2004. Introduction to Machine Learning. © The MIT Press.
Initialize all $Q(s, a)$ arbitrarily
For all episodes
  Initialize $s$
  Repeat
    Choose $a$ using policy derived from $Q$, e.g., $\epsilon$-greedy
    Take action $a$, observe $r$ and $s'$
    Update $Q(s, a)$:
    \[
    Q(s, a) \leftarrow Q(s, a) + \eta (r + \gamma \max_{a'} Q(s', a') - Q(s, a))
    \]
    $s \leftarrow s'$
  Until $s$ is terminal state

Figure 16.5: $Q$ learning, which is an off-policy temporal difference algorithm. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Initialize all \( Q(s, a) \) arbitrarily

For all episodes

1. Initialize \( s \)
2. Choose \( a \) using policy derived from \( Q \), e.g., \( \epsilon \)-greedy

Repeat

1. Take action \( a \), observe \( r \) and \( s' \)
2. Choose \( a' \) using policy derived from \( Q \), e.g., \( \epsilon \)-greedy

Update \( Q(s, a) \):

\[
Q(s, a) \leftarrow Q(s, a) + \eta(r + \gamma Q(s', a') - Q(s, a))
\]

3. \( s \leftarrow s' \), \( a \leftarrow a' \)

Until \( s \) is terminal state

Figure 16.6: Sarsa algorithm, which is an on-policy version of \( Q \) learning. From: E. Alpaydin. 2004. Introduction to Machine Learning. © The MIT Press.
Figure 16.7: Example of an eligibility trace for a value. Visits are marked by an asterisk. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Initialize all $Q(s, a)$ arbitrarily, $e(s, a) \leftarrow 0, \forall s, a$

For all episodes

Initialize $s$

Choose $a$ using policy derived from $Q$, e.g., $\epsilon$-greedy

Repeat

Take action $a$, observe $r$ and $s'$

Choose $a'$ using policy derived from $Q$, e.g., $\epsilon$-greedy

$\delta \leftarrow r + \gamma Q(s', a') - Q(s, a)$

$e(s, a) \leftarrow 1$

For all $s, a$:

$Q(s, a) \leftarrow Q(s, a) + \eta \delta e(s, a)$

$e(s, a) \leftarrow \gamma \lambda e(s, a)$

$s \leftarrow s', a \leftarrow a'$

Until $s$ is terminal state

Figure 16.8: Sarsa($\lambda$) algorithm. From: E. Alpaydın.
Figure 16.9: In the case of a partially observable environment, the agent has a state estimator (SE) that keeps an internal belief state $b$ and the policy $\pi$ generates actions based on the belief states. From: E. Alpaydın. 2004. Introduction to Machine Learning. © The MIT Press.
Appendix: Probability
Figure A.1: Probability density function of $Z$, the unit normal distribution.