CMPE 548 Monte Carlo methods
Assignment 1

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Abstract

In this assignment, you are expected submit one report (hardcopy) includes all results wanted in the questions. All example codes given in this assignment are written in MATLAB.

Exercise 1. (**LLN and CLT**) Suppose we are throwing a loaded coin with an unknown probability π of heads. How many trials would be needed to figure out with 50 percent confidence that indeed the coin is biased, i.e. $\pi \neq 1/2$.

Exercise 2. (**Inversion**) Write a function which samples from a given discrete distribution and given sample size with the inversion method explained in the course. More explicitly, the function takes arguments as follows:

```
samples = yourFunction(discreteDistribution,stateVector,sampleSize)
```

Choose a pretty function name. Denote the discrete distribution as a vector. For instance, for a coin toss, a discrete distribution is [0.5 0.5]. For this case, your states can be `[H T]` (heads and tails). So in this case, if you set $N = 50$, it is expected that the function should return a sequence of `H` or `T` with size 50, distributed according to the discrete distribution. Use a more complicated nonuniform discrete distribution. Your states can be simply natural numbers (or anything). Your report will include,

- Plot of the discrete distribution.
- The histogram (see `hist` function) of your samples.

Use `stem` to plot your discrete distribution (do not use `plot`). You can use `cumsum` function to calculate the CDF while writing your function.

Exercise 3. (**Rejection sampling**) Write a program which samples from a triangular distribution[1] $p(x)$ via the rejection sampling using an appropriate Gaussian proposal $Mq(x)$. You are free to choose the parameters of the triangular distribution, parameters of the Gaussian and M. Make sure that your Gaussian proposal captures the triangular distribution, otherwise your sampling procedure will be wrong. Please try to be efficient as more as possible, i.e., choose a Gaussian proposal which is close enough to your triangular distribution. Add the following things to your report,

[1] See `http://en.wikipedia.org/wiki/Triangular_distribution`

- A plot of the $p(x)$ and $Mq(x)$ so as to show that the proposal captures the $p(x)$.
- A plot of the histogram of your samples (again with `hist` function), so we can see if your samples distributed according to triangular distribution.