Boğaziçi University, Dept. of Computer Engineering

CMPE 482, NUMERIC LINEAR ALGEBRA AND ITS APPLICATIONS

Spring 2015, Final

Name: _____

Student ID: _____

Signature: _____

- Please print your name and student ID number and write your signature to indicate that you accept the University honour code.
- This is a closed book and close notes exam. However, you may use a single double sided A4 cheat sheet.
- Read each question carefully and **WRITE CLEARLY**. Unreadable answers will not get any credit.
- For each question you do not know the answer and leave blank, you can get %10 of the points, if you write only "I don't know the answer but I promise to think about this question and learn its solution".
- There are **6** questions. Point values are given in parentheses.
- You have **180 minutes** to do all the problems.

Q	1	2	3	4	5	6	Total
Score							
Max	10	15	15	20	20	20	100

Some Octave Functions

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[Q R] = qr(A)           % A == Q*R;       QR Factorization
[Qh Rh] = qr(A,0)       % A == Qh*Rh;     Reduced QR Factorization
[U S V] = svd(A)        % A == U*S*V'     Singular Value Decomposition
[U S V] = svd(A, 0)     % A == U*S*V'     Reduced Singular Value Decomposition
x = R\b                 % x == inv(R)*b  Solves a linear system where R is triangular
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1. On the real line, mark and label all numbers that can be represented **exactly** by a 5-bit floating point system with a 3 bit exponent with radix $\beta = 2$. What is $\epsilon_{\text{machine}}$ for this system?

(10 points)

2. Suppose you are given a polynomial $p(x) = \sum_{i=0}^{N} a_i x^i$. Derive an expression to measure how much a particular root x_j where $p(x_j) = 0$ changes when a particular coefficient a_i is perturbed by an infinitesimal quantity δa_i. What are the relative and absolute condition numbers of the root finding problem?

(15 points)

3. (Small perturbations don't reduce rank) Suppose $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_k$ are the nonzero singular values of a rank k matrix A. Show that, if $\|E\|_2 < \sigma_k$, then we have $\text{rank}(A+E) \geq \text{rank}(A)$. *(15 points)*

4. (Autoregressive model) Suppose we are given N noisy measurements of a time series $y = [y_1, \ldots, y_N]^\top$. For $n = 0, 1, \ldots,$ we postulate an autoregressive model

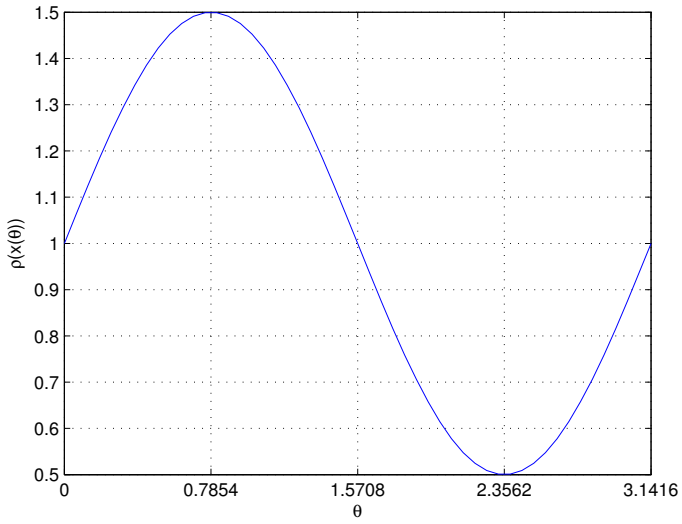
$$y_n \quad = \quad w_1 y_{n-1} + w_2 y_{n-3} + w_3 y_{n-5} + 1 + \epsilon_n$$

   (a) Formulate a problem to find the best $w = [w_1, w_2, w_3]^\top$ that minimizes the norm square of the noise ϵ where $\epsilon = [\epsilon_1, \ldots, \epsilon_N]$.

   (b) Write an Octave program to estimate w given $[y_1, \ldots, y_N]^\top$ and then predict y_{N+1}.

(20 points)

5. Consider the set of 2-vectors $x = x(\theta) = \sqrt{2}\begin{pmatrix} \sin(\theta) \\ \cos(\theta) \end{pmatrix}$ for $0 \leq \theta \leq \pi$. The graph of the function $\rho(x(\theta))x(\theta)^\top A^{-1}x(\theta)$ is given below where A is a real, symmetric and invertible matrix. Hint: $\pi \approx 3.1416$.



(a) What are the eigenvalues and the eigenvectors of A?

(b) What are the eigenvalues and the eigenvectors of $(A - \mu I)^{-1}$ for $\mu \in \mathbb{R}$?

As a function of a and b, state your results. Give justifications for your answers with a short derivation – just writing the answer is not enough.

(20 points)

6. Suppose you are given a polynomial $p(x) = \sum_{i=0}^{N} a_i x^i$ where the coefficients are given as a vector $a = [a_0, a_1, \ldots, a_N]$, where $a_N \neq 0$. Describe a method based on QR factorization to find the smallest (in magnitude) m roots of p. Implement the method in Matlab.

(20 points)