Please print your name and student ID number and write your signature to indicate that you accept the University honour code.

During this examination, you may not use any notes or books.

Read each question carefully and WRITE CLEARLY. Unreadable answers will not get any credit.

For each question you do not know the answer and leave blank, you can get %10 of the points, if you write only “I don’t know the answer but I promise to think about this question and learn its solution”.

There are 5 questions. Point values are given in parentheses.

You have 90 minutes to do all the problems.

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1. What is the output of the following C++ program? For each line numbered from 1-11, write the output. Every step must be explained. (Hint: Be careful with implicit calls to constructors and destructors).

```
#include <iostream>
using namespace std;

template <typename T>
struct obj{
    T i;
    obj(T j=0) : i(j) {cout<<'+';}
    obj(obj<T>& o2) {this->i=o2.i; cout<<'<';}
    ~obj(){cout<<'-';};
    void swap(obj<T> o){
        cout<<"s";
        T temp = i;
        i = o.i;
        o.i = temp;
    }
    T operator/(obj<T>& t){cout<<'/'; return this->i/t.i;}
    T operator/(int j){cout<<'i'; return this->i/j;}
};

template <typename T>
void swap(obj<T> o1, obj<T> o2){
    cout<<"w";
    T temp = o1.i;
    o1.i = o2.i;
    o2.i = temp;
}

int main(int argc, char *argv[]){
    int x=5;
    obj<int> o1(3.7); // ------------------------
    obj<int> o2(5);  // ------------------------
    obj<int> o3(x);  // ------------------------
    swap(o1,o2);    // ------------------------
    cout<<o1.i<<"\n"<<o2.i;  // ------------------------
    o1.swap(o2);    // ------------------------
    cout<<o1.i<<"\n"<<o2.i;  // ------------------------
    cout<<o2/o1;    // ------------------------
    cout<<o3/x;     // ------------------------
    return 0;      // ------------------------
}
```
2. Heaps.

(a) Show the result of inserting 10, 12, 1, 14, 6, 8, 15, 3, 9, 7, 4, 5, 11, 13, 2 into an initially empty ternary heap (one at a time).

[Hint: Ternary heap: a complete tree with the heap ordering property where each node has at most 3 child nodes.]

(b) State the formula to find the positions of the parent and children of an element at position $j$ in a D-heap, where this heap is stored as an array.

[Hint: D-heap: a complete tree with the heap ordering property where each node has at most $D$ child nodes.]

(20 points)
3. Consider a list of classes that students need to take in a curriculum with \( N \) courses. The list you are given consists of some pairs where \((i, j)\) denotes the \textit{prerequisite relation} meaning that course \( i \) must be taken before the course \( j \). Give the pseudocode for an algorithm to test if the curriculum is valid or not, that is a student can finish all courses without violating the prerequisite relation.\hfill (20 points)
4. Minimum spanning tree. For parts (a), and (b) consider the following weighted graph.

(a) Complete the sequence of nodes in the MST in the order that Prim’s algorithm includes them starting from node G.

G ______ ______ ______ ______ ______ ______ ______ ______ ______ ______

(b) Draw the minimum spanning tree and find the total weight.

(20 points)
5. Suppose for a given graph $G_1$, we have computed a minimum spanning tree $T_1$. Now, a new edge to $G_1$ is added. We call this new graph with the added edge $G_2$. Describe an algorithm to compute the minimum spanning tree $T_2$ of $G_2$ efficiently by just adjusting $T_1$.

$(20 \text{ points})$