Boğaziçi University, Dept. of Computer Engineering

CMPE 250, DATA STRUCTURES AND ALGORITHMS

Spring 2012, Final

Name: ____________________________

Student ID: __________________________

Signature: ___________________________

- Please print your name and student ID number and write your signature to indicate that you accept the University honour code.
- During this examination, you may not use any notes or books.
- Read each question carefully and **WRITE CLEARLY**. Unreadable answers will not get any credit.
- For each question you do not know the answer and leave blank, you can get %10 of the points, if you write only "I don't know the answer but I promise to think about this question and learn its solution".
- There are **6** questions. Point values are given in parentheses.
- You have **180 minutes** to do all the problems.

Q	1	2	3	4	5	6	Total
Score							
Max	10	10	10	20	20	30	100

1. Suppose you have algorithms with the five running times listed below. (Assume these are the exact running times.) How much slower do each of these algorithms get when you (i) double the input size, or (ii) increase the input size by one?
   - n^2
   - n^3
   - $100n^2$
   - $n \log n$
   - 2^n

(10 points)

2. Short Definitions (Give concise answers) (2pts each)

   2.1. Express the following statement in big-Oh notation: for some constant C and n_0, $Cf(n)$ is an upper bound of $g(n)$ for $n > n_0$.

   2.2. What is a residual flow graph?

   2.3. What is memoization (Yes, memoization, not memorization)?

   2.4. (4 pts) Fill in the blanks: [(a)______________] is a method for solving a computational problem expressed as a [(b)______________] of steps. Each step can be specified by a list of [(c)______________] that describe a computation. For a given initial state and admissible input, the computations proceed through a well-defined series of successive states and eventually [(d)______________] .
   - (a):
   - (b):
   - (c):
   - (d):

(10 points)

3. Minimum spanning tree. For parts (a), and (b) consider the following weighted graph with 9 vertices and 19 edges. Note that the edge weights are distinct integers between 1 and 19.

   3.1. Complete the sequence of edges in the MST in the order that Kruskal's algorithm includes them.

   `1 ____ ____ ____ ____ ____ ____ ____`

   3.2. Complete the sequence of edges in the MST in the order that Prims's algorithm includes them.

   `6 ____ ____ ____ ____ ____ ____ ____`

(10 points)

4. You are given a directed graph $G = (V, E)$, with a positive integer capacity c_e on each edge e, a designated source $s \in V$, and a designated sink $t \in V$. Suppose you found the maximum $s - t$ flow in G, defined by a flow value f_e on each edge e. Now suppose we pick a specific edge $e \in E$ and increase its capacity by one unit. Explain how to find a maximum flow in the resulting capacitated graph in time $O(m + n)$, where m is the number of edges in G and n is the number of nodes, that is without recomputing the maximum flow from scratch. *(20 points)*

5. Suppose we have n objects corresponding and each object is either of class A or of class B. For each object, we can't directly observe its class but we can compare any two objects and decide if these belong to the same class or not. In other words, for each pair of objects i and j, we label the pair (i, j) either "same" (meaning we believe them both to come from the same class) or "different" (meaning we believe them to come from different classes). We also have the option of rendering no judgment on a given pair, in which case well call the pair ambiguous. So now we have the collection of n objects, as well as a collection of m judgments (either "same" or "different") for the pairs that were not declared to be ambiguous. We would like to know if this data is consistent with the idea that each object is from one of the classes A or B. So more concretely, we'll declare the m judgments to be consistent if it is possible to label each specimen either A or B in such a way that for each pair (i, j) labeled "same," it is the case that i and j have the same label; and for each pair (i, j) labeled "different," it is the case that i and j have different labels. Give an algorithm with running time $O(m + n)$ that determines whether the m judgments are consistent.

(20 points)

6. A certain string-processing language offers a primitive operation which splits a string into two pieces. Since this operation involves copying the original string, it takes n units of time for a string of length n, regardless of the location of the cut. Suppose, now, that you want to break a string into many pieces. The order in which the breaks are made can affect the total running time. For example, if you want to cut a 20-character string at positions 3 and 10, then making the first cut at position 3 incurs a total cost of $20 + 17 = 37$, while doing position 10 first has a better cost of $20 + 10 = 30$. Design a dynamic programming algorithm that, given the locations of m cuts in a string of length n, finds the minimum cost of breaking the string into $m + 1$ pieces.

(30 points)