Boğaziçi University, Dept. of Computer Engineering

CMPE 250, DATA STRUCTURES AND ALGORITHMS

Spring 2011, Final

Name: _____

Student ID: _____

Signature: _____

- Please print your name and student ID number and write your signature to indicate that you accept the University honour code.
- During this examination, you may not use any notes or books.
- Read each question carefully and **WRITE CLEARLY**. Unreadable answers will not get any credit.
- There are **6** questions. Point values are given in parentheses.
- You have **180 minutes** to do all the problems.

Q	1	2	3	4	5	6	Total
Score							
Max	10	15	15	20	20	20	100

1. What is .. (Give short answers. Long answers do not get any credit.)

   1.1. the notation $f(n) = \Theta(g(n))$? (2pts)

   1.2. a data structure ?

   1.3. an algorithm ?

   1.4. a minimum spanning tree?

   1.5. the difference between Kruskal's and Prim's algorithms?

(10 points)

2. The order-of-growth of the running time of one algorithm is $O(N^2)$; the order-of-growth of the running time of a second algorithm is N^3. List **three** compelling (logical, convincing) reasons why a programmer would prefer to use the $O(N^3)$ algorithm instead of the $O(N^2)$ one.

(15 points)

3. In the memory, the following array, corresponding to a binary heap is stored

$$X, T, G, Q, P, F, E, O, M, N, H, A, D, B, C$$

3.1. Is this a max heap, or a min heap ? Draw this heap as a complete tree.

3.2. Show the result of inserting Z.

3.3. Show the result after deleting the maximum element (from the original heap, before Z is inserted).

(15 points)

4. Fill in the following table. (Leave empty if you are unsure as a wrong answer cancels one right answer)

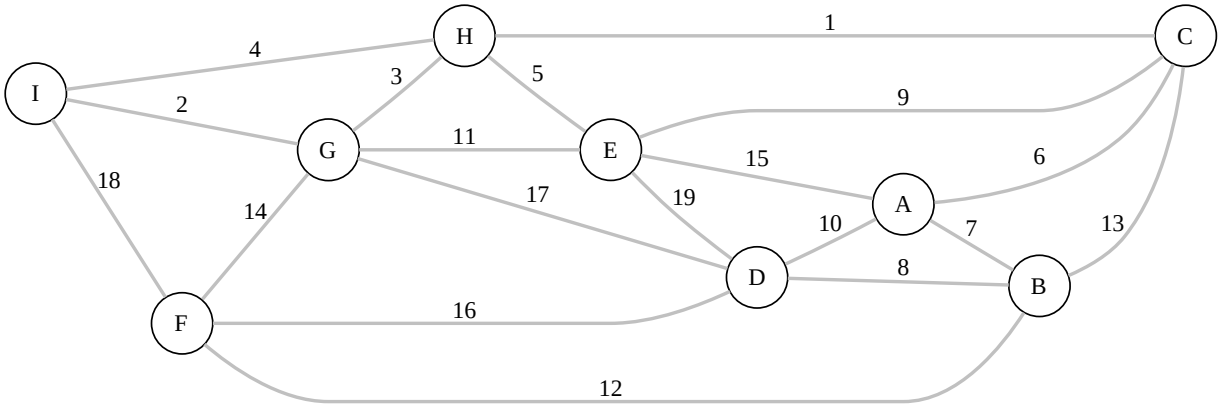
	Insertion Sort	Heapsort	Mergesort	Quicksort
Sequence num				

Below, the column on the left is the original input of strings to be sorted; the column on the right are the string in sorted order; the other columns are the contents at some intermediate step during one of 7 sorting algorithms some of which are listed above. Match up each column by writing its number to the corresponding row labeled as 'sequence'. Use each number exactly once. Briefly, justify your answer.

```
rush abba blue abba fixx abba neyo zman abba
korn acdc cars blue inxs acdc korn yani acdc
fixx blue devo cars korn beck fixx yoyo beck
inxs beck enya devo rush blue inxs tatu blue
cars cars fixx dido cars cake cars styx cake
enya cake fuel enya devo cars enya ween cars
devo devo inxs fixx enya cher devo seal cher
fuel epmd korn fuel fuel devo fuel lons devo
tatu cher moby inxs blue dido lons kiss dido
styx inxs rush korn moby doom mims nofx doom
blue dido styx moby styx enya blue pras enya
moby fuel tatu muse tatu epmd moby rush epmd
abba doom abba rush abba rush abba neyo fixx
muse kiss muse seal dido muse muse muse fuel
seal enya seal styx muse seal cher mims inxs
dido lons dido tatu seal tatu dido fuel kiss
beck fixx beck acdc acdc fixx beck beck korn
kiss neyo kiss beck beck kiss kiss inxs lons
acdc korn acdc doom kiss korn acdc acdc mims
yani moby yani kiss yani yani epmd cars moby
nofx muse nofx nofx doom nofx nofx korn muse
doom pras doom pras nofx styx doom doom neyo
pras mims pras yani pras pras pras blue nofx
yoyo seal yoyo yoyo yoyo yoyo cake moby pras
ween nofx ween ween cake ween rush fixx rush
zman tatu zman zman neyo zman zman abba seal
neyo rush neyo neyo ween neyo ween enya styx
cake yani cake cake zman inxs yoyo cake tatu
epmd ween epmd epmd cher moby yani epmd ween
cher zman cher cher epmd fuel seal cher yani
mims styx mims mims lons mims styx devo yoyo
lons yoyo lons lons mims lons tatu dido zman
-----------------------------------------
 U    1    2    3    4    5    6    7    S
```

(20 points)

5. Minimum spanning tree. For parts (a), and (b) consider the following weighted graph with 9 vertices and 19 edges. Note that the edge weights are distinct integers between 1 and 19.



5.1. Complete the sequence of edges in the MST in the order that Kruskal's algorithm includes them.

1 ____ ____ ____ ____ ____ ____ ____

5.2. Complete the sequence of edges in the MST in the order that Prim's algorithm includes them. Start Prim's algorithm from vertex A.

6 ____ ____ ____ ____ ____ ____ ____

5.3. Given a minimum spanning tree T of a weighted graph G, describe an $O(V)$ algorithm for determining whether or not T remains a MST after an edge $x \leftrightarrow y$ of weight w is added.

(20 points)

6. Let $G = (V, E)$ be a connected, undirected graph with edge-weight function $w : E \to \mathbb{R}$, and assume all edge weights are distinct. Consider a cycle $(v_1, v_2, \ldots, v_k, v_{k+1})$ in G, where $v_{k+1} = v_1$, and let $e^* = (v_i, v_{i+1})$ be the edge in the cycle with the largest edge weight.

   6.1. Plot an example graph that illustrates this scenario

   6.2. Prove that e^* does not belong to the minimum spanning tree T of G. (Assume a spanning tree that has e^* and argue that this can not be a minimum spanning tree.)

(20 points)