Please print your name and student ID number and write your signature to indicate that you accept the University honour code.

During this examination, you may not use any notes or books.

Read each question carefully and WRITE CLEARLY. Unreadable answers will not get any credit.

There are 5 questions. Point values are given in parentheses.

You have 100 minutes to do all the problems.
1. Heap Data Structures. Consider a min-heap storing the values 1, 2, 3, ..., 15. Answer each of the following questions, and justify your answers.

(a) Where in the heap can the value 1 possibly go?
(b) Which values can possibly be stored in entry A[2]?
(c) Where in the heap can the value 15 possibly go?
(d) Where in the heap can the value 6 possibly go?

(20 points)
2. What is the output of the following C++ program? For each line numbered from 1-11, write the output. Every step must be explained. (Hint: Be careful with implicit calls to constructors and destructors).

```cpp
#include <iostream>
using namespace std;

template <typename T>
struct obj{
    T i;
    obj(T j=0) : i(j) {cout<<'+';}
    obj(obj<T>& o2){this->i=o2.i; cout<<'.'<};
    ~obj(){cout<<'-'};
    obj& operator=(obj<T>& o2){this->i=o2.i; cout<<'='; return o2;};
    T operator/>(obj<T>& o2){cout<<'/'; return this->i/o2.i;};
    T operator/>(int j){cout<<'"/"'; return this->i/j;};
};

template <typename T>
void fun1(obj<T>& o){o.i=1; cout<<'1'; return;}
void fun2(obj<int> o){o.i=2; cout<<'2';}
template <typename T>
void fun2(obj<T> o){o.i=3; cout<<'3';}

int main(){
    1    obj<int> o; Output:
    2    obj<double> p(2); Output:
    3    fun1(o); cout<<o.i; Output:
    4    fun2(o); cout<<o.i; Output:
    5    obj<int> o2=o; Output:
    6    obj<int> o3(o); Output:
    7    o2 = o; Output:
    8    cout << o.i/p.i; Output:
    9    cout << o/o.1/2; Output:
    10   cout << p/p; Output:
    11   return 0; Output:
}
```

(20 points)
3. Fill in the following table. (Leave empty if you are unsure as a wrong answer cancels one right answer)

<table>
<thead>
<tr>
<th>Sequence num</th>
<th>Insertion Sort</th>
<th>Heapsort</th>
<th>Mergesort</th>
<th>Quicksort</th>
</tr>
</thead>
</table>

Below, the column on the left is the original input of strings to be sorted; the column on the right are the string in sorted order; the other columns are the contents at some intermediate step during one of 7 sorting algorithms some of which are listed above. Match up each column by writing its number to the corresponding row labeled as 'sequence'. Use each number exactly once. Briefly, justify your answer.

```
rush abba blue abba fixx abba neyo zman abba
korn acdc cars blue inxs acdc korn yani acdc
fixx blue devo cars korn beck fixx yoyo beck
inxs beck enya devo rush blue inxs tatu blue
cars cars fixx dido cars cake cars styx cake
enya cake fuel enya devo cars enya ween cars
devo devo inxs fixx enya cher devo seal cher
fuel epmd korn fuel fuel devo fuel lons devo
tatu cher moby inxs blue dido lons kiss dido
styx inxs rush korn moby doom mims nofx doom
blue dido styx moby styx enya blue pras enya
moby fuel tatu muse tatu epmd moby rush epmd
abba doom abba rush abba rush abba neyo fixx
muse kiss muse seal dido muse muse muse fuel
seal enya seal styx muse seal cher mims inxs
dido lons dido tatu seal tatu dido fuel kiss
beck fixx beck acdc acdc fixx beck beck korn
kiss neyo kiss beck beck kiss kiss inxs lons
acdc korn acdc doom kiss korn acdc acdc mims
yani moby yani kiss yani yani epmd cars moby
nofx muse nofx nofx doom nofx nofx korn muse
doom pras doom pras nofx styx doom doom neyo
pras mims pras yani pras pras pras blue nofx
yoyo seal yoyo yoyo yoyo yoyo cake moby pras
ween nofx ween ween cake ween rush fixx rush
zman tatu zman zman neyo zman zman abba seal
neyo rush neyo neyo ween neyo ween enya styx
cake yani cake cake zman inxs yoyo cake tatu
epmd ween epmd epmd cher moby yani epmd ween
cher zman cher cher epmd fuel seal cher yani
mims styx mims mims lons mims styx devo yoyo
lons yoyo lons lons mims lons tatu dido zman
```

<table>
<thead>
<tr>
<th>U</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>S</th>
</tr>
</thead>
</table>

(20 points)
4. Minimum spanning tree. For parts (a), and (b) consider the following weighted graph with 9 vertices and 19 edges. Note that the edge weights are distinct integers between 1 and 19.

(a) Complete the sequence of edges in the MST in the order that Kruskal’s algorithm includes them.

1  ____  ____  ____  ____  ____  ____  ____  ____

(b) Complete the sequence of edges in the MST in the order that Prim’s algorithm includes them. Start Prim’s algorithm from vertex A.

6  ____  ____  ____  ____  ____  ____  ____  ____

(c) Given a minimum spanning tree $T$ of a weighted graph $G$, describe an $O(V)$ algorithm for determining whether or not $T$ remains a MST after an edge $x \leftrightarrow y$ of weight $w$ is added.

(20 points)
Run Dijkstra’s algorithm on the weighted digraph below, starting at vertex A.

(a) List the vertices in the order in which the vertices are dequeued (for the first time) from the priority queue and give the length of the shortest path from A.

vertex: A C ___ ___ ___ ___ ___ ___ ___
distance: 0 4 ___ ___ ___ ___ ___ ___ ___

(b) Draw the edges in the shortest path tree with thick lines in the figure above.

(20 points)