Boğaziçi University, Dept. of Computer Engineering

CMPE 250, DATA STRUCTURES AND ALGORITHMS

Fall 2010, Final

Name: ______________________________

Student ID: ___________________________

Signature: ____________________________

- Please print your name and student ID number and write your signature to indicate that you accept the University honour code.
- During this examination, you may not use any notes or books.
- Read each question carefully and **WRITE CLEARLY**. Unreadable answers will not get any credit.
- You must get at least 10 points from Q.1 (basic definitions) to get your final graded.
- There are **6** questions. Point values are given in parentheses.
- You have **180 minutes** to do all the problems.

Q	1	2	3	4	5	6	Total
Score							
Max	15	10	10	15	15	15	80

1. What is .. (Give short answers. Long answers do not get any credit.)

   1.1. the notation $O(f(n)) = g(n)$? (1pt)

   1.2. the notation $\Theta(f(n)) = g(n)$? (1pt)

   1.3. the notation $o(f(n)) = g(n)$? (1pt)

   1.4. a Stack ? (1pt)

   1.5. a Priority Queue ? (1pt)

   1.6. a Dequeue ? (1pt)

   1.7. the postfix expression for $(a + b) * (-a) + c/g$? (1pt)

   1.8. the prefix expression for $abcd + *-$? (2pts)

   1.9. a Catalan number ? (1pt)

   1.10. a Bell number ? (1pt)

   1.11. the meaning of the expression `int* e;` in C++ ? (1pt)

   1.12. a data structure ?
   A mathematical object that represents the organization of data, stored on a storage medium such as RAM or disk.

   1.13. an algorithm ?

1.14. a hash function ?

1.15. a heap ?

1.16. a graph ?

1.17. a sparse graph ?

1.18. a minimum spanning tree?

1.19. Kruskal's algorithm?

1.20. a Depth first search ?

1.21. a network problem flow ?

1.22. augmenting path ?

1.23. the dual problem of the maximum flow problem ?

1.24. a pivot (in the context of quicksort) ?

1.25. What is the lower bound complexity of any sort algorithm ?

(25 points)

2. Consider a perfect (complete) binary tree T of height h

   2.1. How many nodes does T contain?

   2.2. What is the average depth of T?
   [Hint: A perfect binary tree of height 1 is $\circ \leftarrow \circ \rightarrow \circ$]

   2.3. Using this result, show that buildheap takes $O(N)$ times where N is the number of elements in an array.

(15 points)

3. Fill in the following table. (Leave empty if you are unsure as a wrong answer cancels one right answer)

	Insertion Sort	Heapsort	Mergesort	Quicksort
Worst case time complexity				
Average case time complexity				
In place? (yes/no)				
Stable? (yes/no)				
Sequence num (2pts each)				

Below, the column on the left is the original input of strings to be sorted; the column on the right are the string in sorted order; the other columns are the contents at some intermediate step during one of the 4 sorting algorithms listed above. Match up each column by writing its number to the corresponding row labeled as 'sequence'. Use each number exactly once.

[Hint: In place: Do we need extra storage other than a few temporary variables of size O(1)? Stable: if two keys are the same, is their original order in the unsorted array guaranteed to be kept after sorting?]

```
COS ARC ARC CHE REL ARC
PHY CHE CHE COS PHY ART
ELE COS COS CHM PHY CEE
COS COS COS COS ELE CHE
MAT ECO ECO COS PHI CHM
MOL ELE EEB ART ORF COS
LIN GEO ELE CEE ORF COS
ARC LIN ELE ARC COS COS
ECO MAE ENG COS ELE COS
CHE MAT GEO COS EEB COS
MAE MOL LIN MAE MUS COS
GEO PHY MAE GEO GEO ECO
ORF ORF MAT ORF ORF EEB
EEB EEB MOL EEB MAT EEB
ENG ENG ORF ENG LIN ELE
ELE ELE PHY ELE COS ELE
COS COS ART ECO COS ELE
ELE ELE CEE ELE ECO ENG
CEE CEE COS LIN CEE GEO
EEB EEB EEB EEB CHE LIN
ART ART ELE MOL ART MAE
MUS MUS MUS MUS MAT MAT
PHI PHI ORF PHI MAE MAT
ORF ORF PHI ORF ELE MOL
COS COS COS MAT COS MUS
PHY PHY PHY PHY MOL ORF
COS COS COS COS COS ORF
MAT MAT MAT MAT EEB ORF
CHM CHM CHM ELE CHM PHI
ORF ORF ORF ORF ENG PHY
COS COS COS PHY COS PHY
REL REL REL REL ARC REL
-----------------------
 U   1   2   3   4   S
```

(24 points)

4. 4.1. what is a template in C++?

   4.2. what is a copy constructor?

   4.3. the output of the following code segment C++ ? Explain

```
char a = 'a'; char& c=a; c = 'c'; cout << 'a' << a << 'c' << c;
```

   4.4. What are the dynamic memory allocation and deallocation operators in C++?

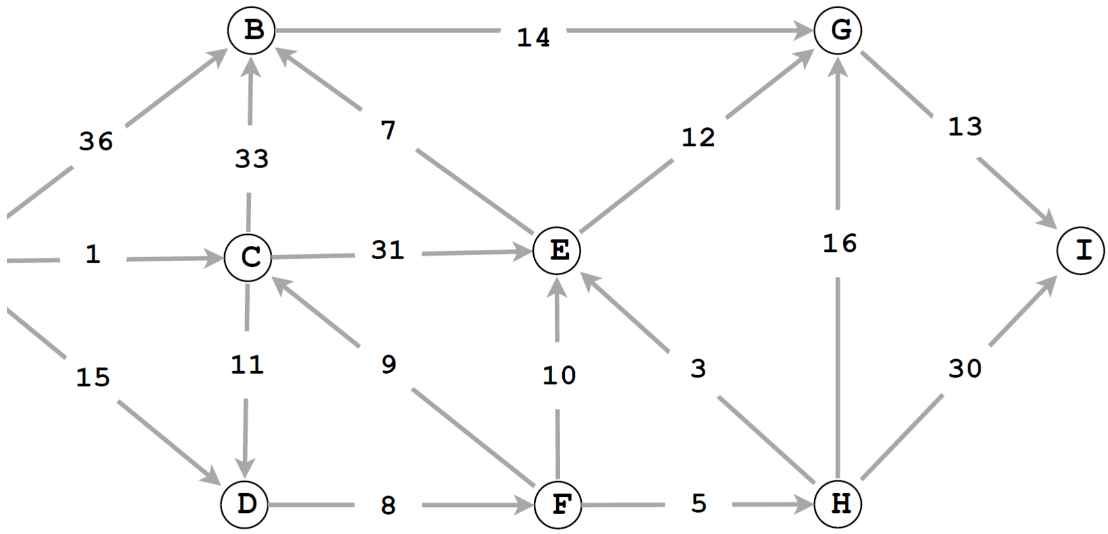
   4.5. What are the dynamic memory allocation and deallocation functions in C?

   4.6. What is the output of the following C++ program? For each line numbered from 1-6, write the output and explain. (Hint: Be careful with implicit calls to constructors and destructors).

```
struct obj {
        int i;
        obj() {cout << '+';};
        obj(obj& o2) {this->i=o2.i; cout << '<';};
        ~obj() {cout << '-';};
        obj& operator=(obj& o2) {this->i=o2.i; cout << '=';return o2;};
};
void fun1(obj& o) {o.i=1; cout << '1'; return;}
void fun2(obj o) {o.i=2; cout << '2'; return;}

   int main() {

1       obj o;                        Output:

2       fun1(o); cout << o.i;         Output:

3       fun2(o); cout << o.i;         Output:

4       obj o2 = o;                   Output:

5       o2 = o;                       Output:

6       return 0;                     Output:
  }
```

(15 points)

5. Run Dijkstra's algorithm on the weighted digraph below, starting at vertex A.



5.1. List the vertices in the order in which the vertices are dequeued (for the first time) from the priority queue and give the length of the shortest path from A.

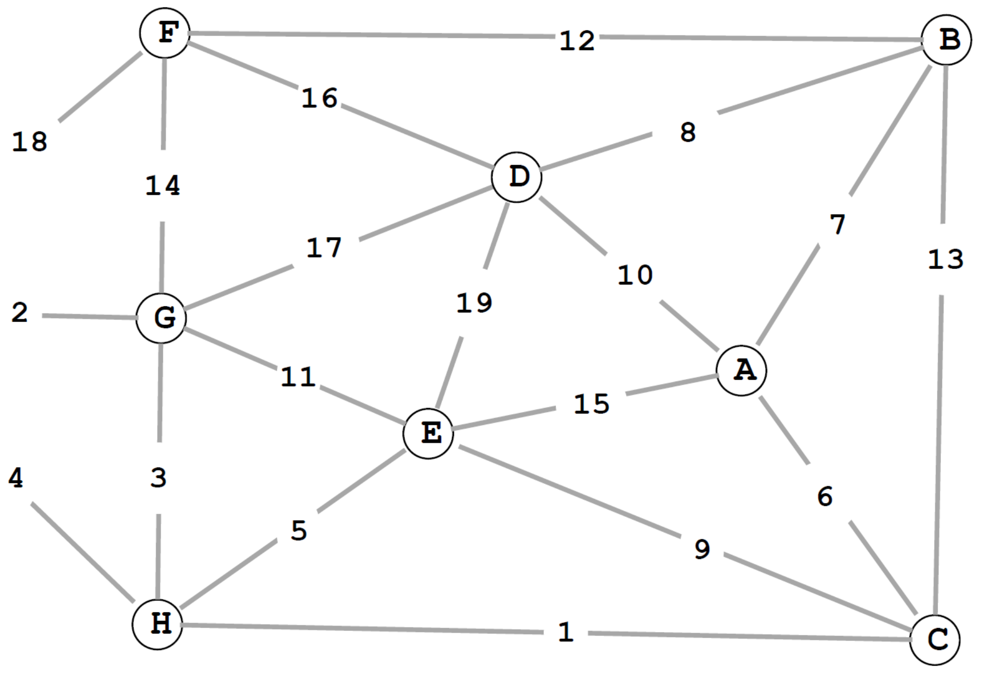
```
vertex:   A C ___ ___ ___ ___ ___ ___ ___
distance: 0 1 ___ ___ ___ ___ ___ ___ ___
```

5.2. Draw the edges in the shortest path tree with thick lines in the figure above.

(15 points)

6. Minimum spanning tree.

For parts (a), and (b) consider the following weighted graph with 9 vertices and 19 edges. Note that the edge weights are distinct integers between 1 and 19.



6.1. Complete the sequence of edges in the MST in the order that Kruskal's algorithm includes them.

`1 ____ ____ ____ ____ ____ ____ ____`

6.2. Complete the sequence of edges in the MST in the order that Prim's algorithm includes them. Start Prim's algorithm from vertex A.

`6 ____ ____ ____ ____ ____ ____ ____`

6.3. Given a minimum spanning tree T of a weighted graph G, describe an $O(V)$ algorithm for determining whether or not T remains a MST after an edge x-y of weight w is added.