**CMPE 300 ANALYSIS OF ALGORITHMS** 29.12.2016

###### FINAL

1. Consider the problem of multiplying *n* numbers; i.e. we want to calculate X1 \* X2 \* ... \* Xn. Write in pseudocode a parallel algorithm on 2-dimensional mesh architecture for this problem. The algorithm must be *cost order optimal* (i.e. cost optimal with respect to the θ class). Assume that the elements X1, X2, ..., Xn have been distributed to the processors initially (an element cannot be in more than one processor).

Analyze the complexity, speedup, cost, and efficiency of the algorithm *clearly*. Explain why it is cost order optimal.

*Note*: No points will be given if the algorithm is not cost order optimal.

1. We are given a set *S* of *n* numbers. We want to test whether an operator ο is associative or not on this set S. That is, we want to test whether (xοy)οz = xο(yοz) for all *x*,*y*,*z*∈*S*.

We have the following assumption: If ο is not associative on a set of *n* numbers, then the operations (xοy)οz and xο(yοz) give the same result for at most *k* triplets *x*,*y*,*z* (1≤*k*≤*n*).

Write in pseudocode a false-biased Monte Carlo algorithm that, given as input a set of *n* numbers, determines whether the operator ο is associative on this set or not. (Do not use any repetition algorithm.) The complexity class of your algorithm must be better than the complexity class of the deterministic algorithm.

Explain *clearly* i) why it is a Monte Carlo algorithm, ii) why it is false-biased, iii) the correctness probability, and iv) the complexity.

1. We are given a group of *n* coins, in which there is one (exactly one) counterfeit (“*sahte*”) coin. A counterfeit coin is lighter than the other coins. We want to find the counterfeit coin. We have a pan balance (“*iki kefeli terazi*”) that we can use to weigh the coins. The pan balance operates in the usual way: when two groups of coins (with W1 the total weight of the coins in one group, W2 the total weight of the coins in the other group) are put onto the two pans, it shows whether W1>W2, W1=W2, or W1<W2. (The weight operation is the basic operation.)

Prove the lower bound of this problem by using *adversary arguments*. Explain clearly.

*Note*: Be careful of not using a particular algorithm or logic. You must use the adversary technique. That is, state clearly what the adversary does during the execution.

*Notes:*

* Where pseudocode is required, the syntax of the pseudocode must be strictly followed. No points will be given if the syntax is not followed or any other language (e.g. C) is used.
* Questions 1,3:35 points; 2:30 points
* Time: 1:30 hours
* Close notes and books

Ne vakit bir yaşamak düşünsem

Bu kurdlar sofrasında belki zor

Ayıpsız fakat ellerimizi kirletmeden

Ne vakit bir yaşamak düşünsem

Sus deyip adınla başlıyorum

İçimsıra kımıldıyor gizli denizlerin

Hayır başka türlü olmayacak

Ben sana mecburum bilemezsin