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3.18 Show that a language is decidable iff some enumerator enumerates the language in lexicographic order.

3.19 Show that every infinite Turing-recognizable language has an infinite decidable subset.

4.10 INFINITE_{DFA} = \{ \langle A \rangle \mid \text{is a DFA and } L(A) \text{ is an infinite language} \}. Show that INFINITE_{DFA} is decidable.

4.21 Let \( S = \{ < M > \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w \} \). Show that \( S \) is decidable.

4.24 A useless state in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

- Prove that there exists a Turing machine \( M \) whose language \( L \) is decidable, but \( M \) is not a decider. This shows that just because a Turing machine’s language is decidable, it’s not necessarily the case that the Turing machine itself must be a decider.

- Let \( L \) be the language of all Turing machine descriptions \( \langle M \rangle \) such that there exists some input on which \( M \) makes at least 5 moves. Show that \( L \) is decidable.