Chapter 4

4.9 INFINITE$_{DFA} = \{ \langle A \rangle \mid \text{is a DFA and } L(A) \text{ is an infinite language} \}$. Show that INFINITE$_{DFA}$ is decidable.

4.18 Let $A$ and $B$ be two disjoint languages. Say that language $C$ separates $A$ and $B$ if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

4.27 Let $C_{CFG} = \{ \langle G, k \rangle \mid L(G) \text{ contains exactly } k \text{ strings where } k > 0 \text{ or } k = \infty \}$. Show that $C_{CFG}$ is decidable.

4.28 Let $A$ be a Turing-recognizable language consisting of descriptions of Turing machines, $\{ \langle M_1, M_2, \ldots \rangle \}$, where every $M_i$ is a decider. Prove that some decidable language $D$ is not decided by any decider $M_i$ whose description appears in $A$. (Hint: You may find it helpful to consider an enumerator for $A$.)

- Show that the language $\{ < M, w, q > \mid |M| \}$ is a Turing Machine that visits state $q$ during its execution when started with input string $w$ is undecidable.

- If a language $L$ is a Turing recognizable but not decidable, then any TM which recognizes $L$ must fail to halt for infinitely many input strings.

- Given an example of a language $L$ such that $L$ is co-Turing recognizable but its complement is not.

- Let $L$ be the language of all Turing machine descriptions $\langle M \rangle$ such that there exists some input on which $M$ makes at least 5 moves. Show that $L$ is decidable.