1. Prove or disprove that regular languages are closed under infinite union.

2. Prove or disprove that regular languages are closed under infinite intersection.

3. Prove or disprove that regular languages are closed under set difference.

4. TRUE or FALSE

(a) If \( L_1 \cup L_2 \) is regular and \( L_1 \) is finite, then \( L_2 \) is regular.

(b) If \( L_1 \cup L_2 \) is regular and \( L_1 \) is regular, then \( L_2 \) is regular.

(c) If \( L_1 \) is regular and \( L_2 \subseteq L_1 \), then \( L_2 \) is regular.

(d) If \( L_1 \) is regular and \( L_2 \) is not regular, then \( L_1 \cup L_2 \) is not regular.

(e) If \( L_1 \) is regular and \( L_1 \cup L_2 \) is not regular, then \( L_2 \) is not regular.

(f) If \( L_1 \) is regular and \( L_2 \) is not regular, then \( L_1 \cup L_2 \) is not regular.

5. \( 1.29 \) Use the pumping lemma to show that the following languages are not regular.

(b) \( A_n = \{www | w \in \{a, b\}^*\} \)

6. \( 1.46 \) Prove that the following regular languages are not regular. You may use the pumping lemma and the closure properties of the class of regular languages under union, intersection and complement.

(a) \( L = \{0^m1^0| n, m \geq 0\} \)

(c) \( L = \{w | w \in \{0, 1\}^*\} \)

(d) \( L = \{w^t w | t \in \{0, 1\}^*\} \)

7. \( 1.54 \) Consider the language \( F = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1, \text{ then } j = k \}

(a) Show that \( F \) is not regular.

(b) Show that \( F \) acts like a regular language in the pumping lemma. In other words give a pumping length \( p \) and demonstrate that \( F \) satisfies the three conditions of the pumping lemma for this value of \( p \).

(c) Explain why parts (a) and (b) do not contradict the pumping lemma.

8. Show that \( L = \{a^{2^n} | n \geq 0\} \) is not regular.

9. TRUE or FALSE

(a) Union of two non-regular languages is always non-regular.

(b) Intersection of two non-regular languages is always non-regular.