• Prove that there exists a Turing machine $M$ whose language $L$ is decidable, but $M$ is not a decider.

3.11 A Turing machine with doubly infinite tape is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.

3.12 A Turing machine with left reset is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, \text{RESET}\}.$$  

If $\delta(q, a) = (r, b, \text{RESET})$, when the machine is in state $q$ reading an $a$, the machine’s head jumps to the left-hand end of the tape after it writes $b$ on the tape and enters state $r$. Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

3.15 Show that the collection of decidable languages is closed under the operation of

a) union.
b) concatenation.
c) star.
d) complementation.
e) intersection.

3.16 Show that the collection of Turing-recognizable languages is closed under the operation of

a) union.
b) concatenation.
c) star.
d) intersection.

d) intersection.

3.18 Show that a language is decidable iff some enumerator enumerates the language in the standard string order.