PS 2 - 19.02.18

1.7 Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts the alphabet is \{0, 1\}.

   b) \{w | w contains the substring 0101 i.e. \(w = x0101y\) for some \(x\) and \(y\)\}

   c) \{w | w contains an even number of 0s or contains exactly two 1s\}

1.12 Let \(D = \{w | w\) contains an even number of a’s and an odd number of b’s and does not contain the substring \(ab\}\). Give a DFA with five states that recognizes \(D\) and a regular expression that generates \(D\). (Suggestion: Describe \(D\) more simply.)

1.14 a) Show that if \(M\) is a DFA that recognizes language \(B\), swapping the accept and nonaccept states in \(M\) yields a new DFA recognizing the complement of \(B\). Conclude that the class of regular languages is closed under complement.

   b) Show by giving an example that if \(M\) is an NFA that recognizes language \(C\), swapping the accept and nonaccept states in \(M\) doesn’t necessarily yield a new NFA that recognizes the complement of \(C\). Is the class of languages recognized by NFAs closed under complement? Explain your answer.

1.18 Give regular expressions generating the languages of Exercise 1.6.

   b) \{w | w contains at least three 1s\}

   g) \{w | w the length of \(w\) is at most 5\}

   l) \{w | w contains at least two 0s and at most one 1\}

1.20 For each of the following languages, give two strings that are members and two strings that are not members—a total of four strings for each part. Assume the alphabet \(\Sigma = \{a, b\}\) in all parts.

   b) \((ba)^*b\)

   c) \(a^* \cup b^*\)

   e) \(\Sigma^* a\Sigma^* b\Sigma^* a\Sigma^*\)

   h) \((a \cup ba \cup bb)\Sigma^*\)

1.31 For any string \(w_1 w_2 \ldots w_n\), the reverse of \(w\), written \(w^R\), is the string \(w\) in reverse order, \(w_n \ldots w_2 w_1\). For any language \(A\), let \(A^R = \{w^R | w \in A\}\). Show that if \(A\) is regular, so is \(A^R\).
Recall that string $x$ is a prefix of string $y$ if a string $z$ exists where $xz = y$, and that $x$ is a proper prefix of $y$ if in addition $x \neq y$. In each of the following parts, we define an operation on a language $A$. Show that the class of regular languages is closed under that operation.

b) $\text{NOEXTEND}(A) = \{ w \in A | w$ is not the proper prefix of any string in $A \}$.

For languages $A$ and $B$, we define the Even operation as $\text{Even}(A, B) = \{ w = uv | u \in A, v \in B$ and $|w| = 2k$ for some $k \geq 1 \}$.

Is the set of regular languages closed under the Even operation? Prove your answer.