4.10 $\text{INFINITEDFA} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ is an infinite language} \}$. Show that $\text{INFINITEDFA}$ is decidable.

4.21 Let $S = \{ \langle M \rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w \}$. Show that $S$ is decidable.

4.24 A useless state in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

- Prove that there exists a Turing machine $M$ whose language $L$ is decidable, but $M$ is not a decider. This shows that just because a Turing machine’s language is decidable, it’s not necessarily the case that the Turing machine itself must be a decider.

- Let $L$ be the language of all Turing machine descriptions $\langle M \rangle$ such that there exists some input on which $M$ makes at least 5 moves. Show that $L$ is decidable.

- Disprove: Every countable language is decidable.