4.20 Let $A$ and $B$ be two disjoint languages. Say that language $C$ separates $A$ and $B$ if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

4.30 Let $A$ be a Turing-recognizable language consisting of descriptions of Turing machines, $\{(M_1), (M_2), \ldots\}$, where every $M_i$ is a decider. Prove that some decidable language $D$ is not decided by any decider $M_i$ whose description appears in $A$. (Hint: You may find it helpful to consider an enumerator for $A$.)

• Given an example of a language $L$ such that $L$ is co-Turing recognizable but its complement is not.

• Prove that the language $\{< M, w, q > | M$ is a Turing machine which visits state $q$ during its execution when started with input string $w\}$ is undecidable.

• Show that the set of undecidable languages are closed under complementation.

• Prove: A language is Turing recognizable iff there exists an enumerator which enumerates it such that every string in the language appears only once in the listing.

• Disprove: Every countable language is decidable.