CmpE 300 - 1. ASSIGNMENT SOLUTIONS

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Question 1 (50 pts.): $\mathcal{F}$ denotes the set of real-valued functions $f(n) : \mathcal{N} \rightarrow \mathcal{R}$ defined on the nonnegative integers that are eventually positive.
Let $f(n)$, $g(n)$, and $h(n) \in \mathcal{F}$. f has a smaller order than g and g has a smaller order than h. Give an example of $f(n)$, $g(n)$, and $h(n)$ for each of the following conditions and show that each condition holds.

Answer:

i. $f(n).h(n) \in o(g^2(n))$
$f(n) = n$
$g(n) = n^3$
$h(n) = n^4$
$f(n).h(n) = n^5$ and $g^2(n) = n^6$
$\lim_{n\to\infty} \frac{f(n).h(n)}{g^2(n)} = \lim_{n\to\infty} \frac{1}{n} = 0 \Rightarrow f(n).h(n) \in o(g^2(n))$

ii. $f(n).h(n) \in \theta(g^2(n))$
$f(n) = 2n^2$
$g(n) = n^3$
$h(n) = n^4$
$f(n).h(n) = 2n^6$ and $g^2(n) = n^6$
$\lim_{n\to\infty} \frac{f(n).h(n)}{g^2(n)} = \lim_{n\to\infty} \frac{2}{1} = 2 \Rightarrow f(n).h(n) \in \theta(g^2(n))$

iii. $g^2(n) \in o(f(n).h(n))$
$f(n) = 2n^2$
$g(n) = n^3$
$h(n) = n^7$
$f(n).h(n) = 2n^9$ and $g^2(n) = n^6$
$\lim_{n\to\infty} \frac{f(n).h(n)}{g^2(n)} = \lim_{n\to\infty} \frac{2n^3}{1} = \infty \Rightarrow g^2(n) \in o(f(n).h(n))$

Question 2 (50 pts.): What is the running time of the following function $modthree(n)$ in terms of Ω, Θ, and O notation? Note that you will use a single complexity function for all values of n; not different functions for different values. (i.e. $f(n) = ..., n \in \mathcal{N}$).

```
function modthree(n)
m = 0
if n mod 3 = 0
    for i = 1 to n * n do
        m = m + 2
    endfor
elseif n mod 3 = 1
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```
    for i = 1 to n * n * n do
        m = m + 3
    endfor
else
    for i = 1 to n * n * n * n do
        m = m + 4
    endfor
endif
return m
end modthree
```

Answer: We can write the running time of the above function as $n^{2+(n \bmod 3)}$.

i. $n^{2+(n \bmod 3)} \in \Omega(n^2)$

ii. $n^{2+(n \bmod 3)} \in O(n^4)$

iii. $n^{2+(n \bmod 3)}$ takes different values according to value of n such that n^2, n^3, or n^4. So, we cannot say anything about Θ class. Maybe, one can analyse the worst-case running time of the function. Obviously, $W(n^{2+(n \bmod 3)}) \in \Theta(n^4)$.