

CMPE 300 – Fall 2012

Assignment 1 – Due 13/11/2012

Please answer the following questions and submit your answers to your assistant until 17:00 Tuesday, November 13, 2012. The sole purpose of this homework is to familiarize you with the processes involved in answering complexity related questions. Please work on them by your own. Please do not submit just an answer, but show all your reasoning, and how you arrive at the answers. For any further questions, contact the assistant.

1. You are given the following function:

```
function Q1(X[1:n])
Input: X[1:n] array of elements with  $X[i] \in \{0,1\}$ ,  $1 \leq i \leq n$ ,
      n is even
Output: a (an integer)
b = 1
a = 0

for i = 1 to n do
    for j = i to n do
        a = a + X[i]
    endfor
    if X[i]  $\neq$  X[n - i + 1]
        b = 0
    endif
endfor
if b = 1 then
    for i = 1 to n do
        for j = 1 to n do
            a = a + a * X[j]
        endfor
        for j = 1 to  $2^{n/2}$  do
            a = a + 1
        endfor
    endfor
endif
```

Do a fine analysis of the function and express the worst-case $W(n)$ and average case $A(n)$ of the algorithm using Θ notation. You may consider all the assignment statements in the algorithm as the basic operation and the input as a binary string.

2. Prove or disprove: There exist two non-negative functions $f(n)$ and $g(n)$ such that neither $f(n) \in O(g(n))$ nor $g(n) \in O(f(n))$.
3. Given a function $f(n)$, determine whether the following statements are true or not. Justify your answers formally.

$$f(n) = n^4 \log(n!) + \sum_{i=1}^5 i^n$$

- a) $f(n) \in O(n^5 \log n)$
- b) $f(n) \in \Omega(n^5 \log n)$
- c) $f(n) \in \Theta(5^n)$
- d) $f(n) \in o(5^n)$