This is an individual homework. Bring the hard copy of your homeworks to the final exam. Please do not submit just an answer, but show all your reasoning, and how you arrive at the answers. For any further questions, contact the assistant.

Question 1

Let \( L = \{a_1, a_2, \ldots, a_n\} \) be a list of \( n \) integers.

a) Write an EREW algorithm for computing \( a_1 \ast a_2 \ast \cdots \ast a_n \) using \( \frac{n}{2} \) processors. Analyze the complexity of your algorithm. Is your algorithm cost optimal?

b) Write an EREW algorithm for computing \( a_1 \ast a_2 \ast \cdots \ast a_n \) using \( \frac{n}{\log n} \) processors (You may assume that \( \frac{n}{\log n} \) is an integer). Analyze the complexity of your algorithm. Is your algorithm cost optimal?

c) Write a CRCW algorithm for testing whether \( a_1 \ast a_2 \ast \cdots \ast a_n = 0 \). Analyze the complexity of your algorithm. Is your algorithm cost optimal? You will get more or less points depending on the time complexity of your algorithm.

Question 2

Suppose that there are \( n \) students in a class where \( n \) is even. I want to find a student who scored better than half of the students in the class.

a) Determine the lower bound for the worst-case complexity of the problem.

b) Describe a Monte Carlo algorithm for solving the problem which gives a correct answer \( \frac{1}{2} \) of the time. What is the runtime of your algorithm?

c) Modify your algorithm in part b) to obtain an algorithm which gives a correct answer \( \frac{3}{4} \) of the time. What is the runtime of your algorithm?

d) Now generalize the idea in part c) to describe an algorithm which gives a correct answer \( 1 - \frac{k}{2^r} \) of the time for some constant \( k \). What is the runtime of your algorithm?