This is an individual homework. Please answer the following questions and submit your answers to your assistant as hard copy. Please work on them by your own. Please do not submit just an answer, but show all your reasoning, and how you arrive at the answers. It is also expected that you write your full name and student id on the upper left corner of each paper and staple all your papers together. For any further questions, contact the assistant.

**Question 1 (50 Points)**

Suppose that there are 100 software engineers in a software house. In the first month of the year, a performance evaluation is made and bonus is given to the top 25% of software engineers according to their performance score. The bottom 10% of software engineers get no salary increase. The others get standard salary increase.

a) I want to find a software engineer who got a bonus payment. Describe a Monte Carlo algorithm for solving the problem which gives a correct answer \( \frac{1}{4} \) of the time. What is the runtime of your algorithm? **(15 Points)**

b) I want to find a software engineer who got no salary increase. Describe a Monte Carlo algorithm for solving the problem which gives a correct answer \( \frac{19}{100} \) of the time. What is the runtime of your algorithm? **(15 Points)**

c) I want to find a software engineer who got either a standard salary increase or a bonus payment. Describe a Monte Carlo algorithm which gives a correct answer \( 1 - \left( \frac{10}{100} \right)^k \) of the time for some constant \( k \). What is the runtime of your algorithm? **(20 Points)**
Question 2 (50 Points)

Suppose you are in a building with \( k \) floors and suppose you have an infinite number of identical eggs at your disposal. The eggs you are given break if they are dropped from the \( N^{th} \) floor or above. You can drop an egg from a floor in order to test if it breaks. We want to find that \( N \), with minimum number of trials. \( N \leq k \) is given. If an egg breaks, it becomes unusable. (Note: Eggs do not break if dropped from the 0\(^{th} \) floor since they already are on the ground.)

1. Determine the lower bound of ‘number of trials to find \( N \)’ for \( k \) floors. (15 Points)

2. Now assume you have two eggs instead of infinite number of eggs. Determine the lower bound of ‘number of trials to find \( N \)’ for \( k \) floors. Note that you cannot use the same strategy as infinite number of eggs here. You need to be cautious with your eggs so that you find \( N \) no matter what. (Hint: Think of the case when you have only one egg as a starting point, then analyze the situation based on possible outcomes, and extend it.) (25 Points)

3. Solve the problem for \( \infty \) eggs and 1024 floors. (5 Points)

4. Solve the problem for 2 eggs and 500 floors. (5 Points)