1. (pp. 59 Question 12) Let \( I(x) \) be the statement “\( x \) has an Internet connection” and \( C(x, y) \) be the statement “\( x \) and \( y \) have chatted over the Internet”, where the domain for the variables \( x \) and \( y \) consists of all the students in your class. Use quantifiers to express each of these statements.

d) No one in the class has chatted with Bob.

\[ \forall x \neg C(x, \text{Bob}) \]

h) Exactly one student in your class has an Internet connection.

\[ \exists x [I(x) \land \forall y (I(y) \rightarrow x = y)] \]

j) Everyone in your class with an Internet connection has chatted over the Internet with at least one other student in your class.

\[ \forall x [I(x) \rightarrow \exists y (C(x, y) \land x \neq y)] \]

l) There are two students in your class who have not chatted with each other over the Internet.

\[ \exists x \exists y (x \neq y \land \neg C(x, y)) \]

n) There are at least two students in your class who have not chatted with the same person in your class.

Solution
n) There are at least two students in your class who have not chatted with the same person in your class.
\[ \exists x \exists y (x \neq y \land \neg \exists z (C(x, z) \land C(y, z))] \]

2. Given the following premises

i) \( \forall x (\neg K(x) \rightarrow \neg E(x)) \)

ii) \( \neg H(Ahmet) \)

iii) \( T(Mehmet) \)

iv) \( \forall x(K(x) \rightarrow \neg S(x)) \)

v) \( \forall x(\neg H(x) \rightarrow T(x)) \)

vi) \( \forall x(\neg E(x) \leftrightarrow (x = Mehmet)) \)

give a formal proof for the statement \( \neg S(Ahmet) \). Note that "Ahmet" and "Mehmet" are the elements of the domain.

Solution

(1) \( \forall x (\neg K(x) \rightarrow \neg E(x)) \) Premise

(2) \( \neg H(Ahmet) \) Premise

(3) \( T(Mehmet) \) Premise

(4) \( \forall x(K(x) \rightarrow \neg S(x)) \) Premise

(5) \( \forall x(\neg H(x) \rightarrow T(x)) \) Premise

(6) \( \forall x(\neg E(x) \leftrightarrow (x = Mehmet)) \) Premise

(7) \( \neg E(Ahmet) \leftrightarrow (Ahmet = Mehmet) \) (6) Universal Instantiation

(8) \( \neg E(Ahmet) \rightarrow (Ahmet = Mehmet) \) Logical Eq. Table 8 and Simplification in (7)

(9) \( (Ahmet \neq Mehmet) \) Tautology

(10) \( E(Ahmet) \) (8) and (9) modus tollens

(11) \( \neg K(Ahmet) \rightarrow \neg E(Ahmet) \) (1) Universal Instantiation

(12) \( \neg (\neg E(Ahmet)) \) (10) Double negation

(13) \( \neg (\neg K(Ahmet)) \) (11) and (12) Modus Tollens

(14) \( K(Ahmet) \) (13) Double Negation

(15) \( K(Ahmet) \rightarrow \neg S(Ahmet) \) (4) Universal Instantiation

(16) \( \neg S(Ahmet) \) (14) and (15) Modus Ponens

3. (pp. 147 Question 30) If \( f \) and \( f \circ g \) are one-to-one, does it follow that \( g \) is one-to-one? Justify your answer.

Solution

Yes. Proof by Contradiction:
Assume \( f \) and \( f \circ g \) are one-to-one, but \( g \) is not one-to-one. Then there exists \( x \) and
y in the domain of g such that $x \neq y$ and $g(x) = g(y)$. Then, $f(g(x)) = f(g(y))$. This implies $(f \circ g)(x) = (f \circ g)(y)$ which contradicts to the fact that $f \circ g$ is one-to-one. Hence, the assumption is wrong. The statement, if $f$ and $f \circ g$ are one-to-one, then $g$ is one-to-one is correct.