Quiz1 (October 13, 2009 Tuesday)

**Question**

\[ A = \{1, 3, 7\} \text{, } B = \{a, b, c, d\} \text{ and } \alpha = \{(3, b), (3, c), (7, c)\} \text{. Find} \]

(a) \(\alpha^{-1} \circ \alpha\)

(b) \(\alpha \circ \alpha^{-1}\)

**Answer**

Definition of Composition of Relations: Let \(\alpha \subseteq A \times B\), \(\beta \subseteq B \times C\). The composition of \(\alpha\) and \(\beta\), \(\beta \circ \alpha \subseteq A \times C\): \(\beta \circ \alpha \equiv \{(a, c) \in A \times C \mid \exists b \in A[a \alpha b \land b \beta c]\}\)

(a) \(\alpha^{-1} \circ \alpha \subseteq A \times A\) and \(\alpha^{-1} \circ \alpha = \{(3, 3), (3, 7), (7, 3), (7, 7)\}\)

(b) \(\alpha \circ \alpha^{-1} \circ \alpha \subseteq B \times B\) and \(\alpha \circ \alpha^{-1} = \{(b, b), (b, c), (c, b), (c, c)\}\)

Quiz2 (October 27, 2009 Tuesday)

**Question**

Let \(A\) be a set, and \(*\) be a binary operation on \(A\). Define

\[
\begin{align*}
a^1 &= a \\
a^{n+1} &= a^n * a \text{ for } n \in \mathbb{Z}^+ 
\end{align*}
\]

Consider the sequence \(a^1, a^2, a^3, \ldots\). Does the sequence repeat itself. If so why, if not how?

**Answer**

Since \(*\) is a binary operation on \(A\), \(a^n \in A\) for \(n \in \mathbb{Z}\). Hence if \(A\) is a finite set the sequence \(a^1, a^2, a^3, \ldots\) has to repeat itself. If the set \(A\) is infinite, the sequence could still repeat itself if \(a^k = e\) for some \(k \in \mathbb{Z}\)
Quiz3 (7)

Question
Let \([A, \circ]\) be a subgroup of \([B, \ast]\). What can we say about the identity of \(A\) and \(B\).

Answer
Since \([A, \circ]\) be a subgroup of \([B, \ast]\), \(\circ\) is the restriction of \(\ast\) on \(A\). Therefore, their identities must be the same.

Quiz4 (November 24, 2009 Tuesday) Attendance Quiz

Quiz5 (December 14, 2009 Tuesday) Attendance Quiz

Quiz6 (Presentation Quiz)

Question
What is the sum of degrees of the nodes in a tree with \(n\) nodes.

Answer
In a tree with \(n\) nodes there are \(n - 1\) edges. Each edge contributes 2 to the total degree. Hence, the total degree is \(2(n - 1)\).