Novel quantization-based spectrum sensing scheme under imperfect reporting channel and false reports

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SUMMARY

In this paper, a novel sensing scheme, uniform quantization for cooperative sensing (UniQCS), that employs a uniform quantizer is proposed. UniQCS is based on energy detection and uses weight vector for global decision function. It performs better than hard decision algorithms such as majority and $k$-out-of-$n$ in terms of probability of detection and false alarm at the cost of a marginal increase in overhead bits under imperfect reporting channel and false reports. The probability of detection is maximized for a given probability of false alarm constraint by the proposed method. For detailed analysis, the UniQCS is compared with equal gain combiner scheme, which performs far better than hard decision algorithms, via highest bandwidth requirement. The proposed algorithm performs close to equal gain combiner. Moreover, the robustness of UniQCS to sensing error is analyzed when some nodes always report false decisions to the fusion center and the reporting channel is imperfect. For probability of false alarm equal to 0.01, performance gain of UniQCS is at least 45% compared with the other methods when there are two false reporting nodes. UniQCS performance gain is at least 15% compared with other methods for probability of reporting channel error equal to 0.001. Copyright © 2012 John Wiley & Sons, Ltd.

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1. INTRODUCTION

The sharing of local observations between the secondary users and the fusion center is the most crucial factor that determines the performance of cooperative sensing. Detection performance is determined by the quality of local observations and the quality of the information received by fusion center. Therefore, the number of quantization bins, the number of bits sent for sensing reports, and the global decision logic affect the system performance. Furthermore, the imperfections in the reporting channel and the erroneous reports due to malfunctioning or malicious secondary devices should also be considered. Distorting impacts of these factors on the global decision logic must be analyzed for optimizing the sensing performance.

Decreasing the number of bits for sensing reports with acceptable performance enables increasing the number of sensing periods and better performance. Furthermore, having a bandwidth-limited reporting channel does not allow sending the whole observation and using complicated protocols for sending the sensing reports to fusion center. Hence, the nodes should quantize their observations in an optimal manner rather than sending the exact observation values. Using more quantization bins increases the quality of the information sent, at the cost of congestion in the reporting channel.

A comprehensive classification of cooperative sensing is examined in [1], and research challenges are listed. Various cooperative sensing techniques are studied in [2–5]. The effect of user

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collaboration in Rayleigh fading channel is studied in [6]. The authors show that increase in the number of cooperative users results in significant increase in sensing performance and spectrum utilization. In [7], cooperative sensing and quantization schemes (1-bit or hard decision) are investigated for multiple primary bands. The effects of imperfect reporting channel to sensing performance are analyzed for hard decision logic in [5]. The spectrum sensing scheduling and sensing time are analyzed in [8, 9]. In [10], the authors analyze the cooperative sensing under bandwidth constraints. In [11], the authors examine the optimal quantizer for signal detection locally. The work is extended in [12] by using evidence theory-based cooperative spectrum sensing with efficient quantization. Similarly in [13], detection and false alarm probabilities are derived with consideration of errors in the reporting channel due to fading. Only the hard decision logic \(k\)-out-of-\(n\) is studied in the paper. In [14] and [15], cooperative sensing via quantization is studied for 2-bit and 3-bit quantization. However, global decision logic is static in terms of weights. Our work generalizes global decision logic, \(n\)-bit quantization, and improves weights by using genetic algorithm. In [16], bit error probability (BEP) wall is introduced and performance analysis for \(k\)-out-of-\(n\) is performed. Authors analyze the signal-to-noise ratio (SNR) loss due to BEP without considering the overall detection probabilities.

In this paper, a novel cooperative sensing scheme is proposed, which uses uniform quantization named \textit{uniform quantization for cooperative sensing} (UniQCS), and optimizes the parameters of the proposed method to maximize the probability of detection while using a uniform quantizer. Most of the works on quantization focus on just the local quantization process; instead, we optimize the overall performance in terms of cooperative detection. Sensing performance of UniQCS is close to the \textit{equal gain combining} (EGC) (which provides an upper bound for the schemes that do not differentiate the nodes) and better than conventional hard decision algorithms at the cost of a marginal increase in overhead bits. Furthermore, the robustness of UniQCS is verified under imperfect reporting channel and false reports.

The rest of the paper is organized as follows: In Section 2, we overview conventional energy detection algorithms and give details for the evaluation of detection and false alarm probabilities under different channel conditions. We present the proposed method and the cooperative sensing in Section 3 including the evaluation of detection and false alarm probabilities under imperfect reporting channel and false alarms. In Section 4, we present the results for performance analysis of proposed sensing method and robustness against imperfect reporting channel and false reports. We finally reveal the conclusion and future directions in Section 5.

2. BACKGROUND ON SPECTRUM SENSING

In a nutshell, the goal of spectrum sensing is to decide between two hypotheses

\[
\begin{align*}
Y(t) &= n(t) \quad H_0 \text{ (white space)}, \\
Y(t) &= h \times s(t) + n(t) \quad H_1 \text{ (occupied)}
\end{align*}
\]

(1)

where \(Y(t)\) is the complex signal received by the cognitive radio device, \(s(t)\) is the transmitted signal of the primary user, \(n(t)\) is the \textit{additive white Gaussian Noise} (AWGN) and \(h\) is the complex gain of the ideal channel.

Widespread transmitter detection methods in the literature are \textit{energy detection} (ED), \textit{matched filter detection}, and \textit{cyclostationary detection}. ED-based approach, also known as radiometry or periodogram, is the most common way of spectrum sensing in high SNR conditions because it does not require any \textit{a priori} knowledge of primary signals and has much lower computational and implementation complexity [17, 18]. We follow the ED approach due to these advantages.

The conventional energy detector uses single threshold to determine the presence or absence of the signal. The input band-pass filter selects the center frequency, \(f_c\), and the bandwidth of interest, \(W\), in hertz. This filter is followed by a squaring device to measure the received energy and an integrator that determines the observation interval, \(T\), in seconds. Finally, the output is compared with a threshold, \(\lambda\), to decide whether the signal is present. Presence of primary user in AWGN as well as flat fading Rayleigh channels results in different ED outputs.
Under AWGN channel, energy received \( O_i = \sum_{j=1}^{2TW} Y_{ij}^2 \) by secondary user \( i \) follows the distribution

\[
f(O_i | \gamma) \approx \begin{cases} \chi^2_{2TW} & H_0 \\ \chi^2_{2TW}(2\gamma) & H_1 \end{cases}
\]

(2)

where \( \chi^2_{2TW} \) and \( \chi^2_{2TW}(2\gamma) \) are central and non-central chi-squared distributions, respectively [19, 20]. \( TW \) and \( \gamma \) represent the bandwidth product and SNR, respectively. Under AWGN channel conditions, SNR value is fixed.

In the single threshold ED, the decision for \( H_0 \) and \( H_1 \) depends solely on \( \lambda \). Probability of detection, \( P_d^A \), and the probability of false alarm, \( P_f^A \), for a single secondary user under AWGN channel can be calculated using \( \lambda \) with the exact closed-form equations:

\[
P_d^A = P(O_i > \lambda \mid H_1) = Q_{TW}(\sqrt{2\gamma}, \sqrt{\lambda})
\]

(3)

\[
P_f^A = P(O_i > \lambda \mid H_0) = \frac{\Gamma(TW, \lambda/2)}{\Gamma(TW)}
\]

(4)

where \( Q_\alpha \) is the Marcum Q-function, and \( \Gamma(,.) \) and \( \Gamma(,,.) \) represent the gamma and incomplete gamma functions, respectively [19, 20].

The mobile radio channel is characterized by multipath reception. The signal offered to the receiver contains not only a direct line-of-sight radio wave, but also a large number of reflected radio waves [21]. Even worse in urban centers, the line-of-sight is often blocked by obstacles. The basic model of Rayleigh fading assumes received multipath signal to consist of a large number of reflected waves with independent and identically distributed (i.i.d.) in-phase and quadrature amplitudes and not the line-of-sight component. Therefore, we focus on Rayleigh fading channel caused by multipath reception. Under Rayleigh fading channel, the probability of false alarm, \( P_f^R \), remains the same as in the case of AWGN channel because it depends only on the distribution of noise. However, under the Rayleigh channel with no diversity, signal amplitude follows Rayleigh distribution [20]. Therefore, SNR \( \gamma \) follows exponential PDF:

\[
f(\gamma) = \frac{1}{\gamma} \exp(-\gamma/\bar{\gamma}), \quad \gamma \geq 0.
\]

(5)

The average \( P_d \) in this case, \( P_d^R \), can be calculated by averaging Equation (3) over Equation (5).

\[
P_d^R = e^{-\lambda/2} \sum_{n=0}^{TW-2} \frac{1}{n!} \left( \frac{\lambda}{2} \right)^n \left( 1 + \frac{\bar{\gamma}}{\bar{\gamma}} \right)^{TW-1} \left[ e^{-\frac{\lambda}{2(1+\bar{\gamma})}} - e^{-\frac{\lambda}{2}} \sum_{n=0}^{TW-2} \frac{1}{n!} \left( \frac{\lambda \bar{\gamma}}{2(1+\bar{\gamma})} \right)^n \right]
\]

(6)

Under Rayleigh fading channel in which diversity paths are i.i.d., the output SNR, \( \gamma_t \), is the sum of the SNRs on all branches and can be used for evaluating average \( P_d \) for EGC scheme.

The PDF of \( \gamma_t \) for i.i.d. Rayleigh branches is given by

\[
f(\gamma_t) = \frac{1}{(L-1)!\bar{\gamma}^L} (\gamma_t)^{L-1} \exp(-\gamma_t/\bar{\gamma})
\]

(7)

where \( L \) is the number of i.i.d. diversity branches. The PDF in Equation (7) is similar to PDF of SNR in the Nakagami channel. Nakagami parameter \( m \) can be viewed as a diversity order. Hence, the average \( P_d \) for the EGC scheme, \( P_d^E \), is obtained by replacing \( m, \bar{\gamma}, \) and \( TW \) by \( L, L\bar{\gamma}, \) and \( LTW \), respectively [20]. We take diversity order \( L \) equals to the number of cooperating nodes for evaluating \( P_d^E \). Because we do not differentiate the nodes with different SNR levels, EGC constitutes an upper bound for our optimization problems.

\[
P_d^E = \alpha \left[ \Psi + \beta \sum_{n=1}^{LTW-1} \frac{(\lambda/2)^n}{2n!} \right] F_1 \left[ L; n + 1; \frac{\lambda}{2} \frac{\bar{\gamma}}{2(1+\bar{\gamma})} \right]
\]

(8)
where $F_1(\ldots)$ is the confluent hypergeometric function,

$$\alpha = \frac{1}{\Gamma(L)2^{L-1}} \left( \frac{1}{\gamma} \right)^L,$$

$$\beta = \Gamma(L) \left( \frac{2\gamma}{1 + \gamma} \right)^L \exp(-\lambda/2).$$

and

$$\psi = \frac{2^{L-1}(L-1)!}{(1/\gamma)^L(1 + \gamma)} e^{-\lambda/(1+\gamma)} \left[ \left( 1 + \frac{1}{\gamma} \right)^L \left( \frac{1}{1 + \gamma} \right)^{L-1} L_{L-1} \left( -\frac{\lambda\gamma}{2(1 + \gamma)} \right) + \sum_{n=0}^{L-2} \left( \frac{1}{1 + \gamma} \right)^n L_n \left( -\frac{\lambda\gamma}{2(1 + \gamma)} \right) \right]$$

where $L_n(\cdot)$ is the Laguerre polynomial of degree $n$.

These formulations form the basics of the spectrum sensing methods. In Section 3, the proposed method uses these formulations for determining the probability of having observation in the quantization bins under given channel condition and $H_0$ or $H_1$.

3. UNIFORM QUANTIZER FOR COOPERATIVE SENSING (UNIQCS)

The conventional ED is a quantizer with two bins that use single threshold to determine the presence or absence of the signal. By using more quantization bins at the sensing nodes, the information gain accrued by cooperation can be increased. Thresholds divide the observation space into bins and the sensing node determines the bin into which the observation falls.

We propose to quantize the observed energy $O_i$ by secondary user $i$ locally and collect such information from all secondary users at the fusion center and give a global decision in an optimal manner. Most of the works on quantization focus on just the local quantization process. We do not just optimize local quantization process; instead, we optimize the overall performance in terms of cooperative detection. Therefore, the proposed method has two parts: local quantization and global decision logic.

3.1. Local quantization

In Figure 1, quantization levels with four bins are depicted. There are three thresholds that are determined by the first threshold, $\lambda_1$, because the distance between consecutive thresholds is fixed and denoted by $\Delta$ (i.e., uniform quantizer). Observation of node $i$, $O_i$, is greater than or equal to zero, and the observation space is divided into bins where $B_k$ denotes the $k$th bin.

For a given $\lambda_1$, other thresholds can be found by adding $\Delta$ for the next threshold at each step. Hence, the degree of freedom is just one for a given $\Delta$. For a given set of thresholds \{$\lambda_1, \lambda_2, \ldots, \lambda_{n-1}$\}, we can evaluate the probability of having observation $O_i$ in bin $k$ under $H_0$ and $H_1$, respectively. $P_{H_0}^A(B_k)$ denotes the probability of having local observation $O_i$ in bin $B_k$ under hypothesis $H_i$ and AWGN channel:

$$P_{H_0}^A(B_k) = \begin{cases} 1 - G_{TW}(\lambda_k) & \text{if } k = 1 \\ G_{TW}(\lambda_{k-1}) & \text{if } k = n \\ G_{TW}(\lambda_{k-1}) - G_{TW}(\lambda_k) & \text{otherwise} \end{cases}$$

$$P_{H_1}^A(B_k) = \begin{cases} 1 - Q_{TW}(\gamma, \lambda_k) & \text{if } k = 1 \\ Q_{TW}(\gamma, \lambda_{k-1}) & \text{if } k = n \\ Q_{TW}(\gamma, \lambda_{k-1}) - Q_{TW}(\gamma, \lambda_k) & \text{otherwise} \end{cases}$$

where $G_{TW}(\lambda)$ denotes $\frac{\Gamma(TW, \lambda/2)}{\Gamma(TW)}$ and $Q_{TW}(\gamma, \lambda)$ denotes $Q_{TW}(\sqrt{2\gamma}, \sqrt{\lambda})$. 

By using Equation (6), similar formulas can be easily obtained by replacing \( Q_{TW}(\gamma, \lambda) \) with the \( P_d \) formulation for the Rayleigh channel.

### 3.2. Cooperative sensing with decision vector (CSDV)

To take advantage of the spatial diversity in the wireless channel, cooperative spectrum sensing methods have been proposed in [22–24]. We provide a cooperative sensing scheme for similar reasons.

The system model for the proposed method is depicted in Figure 1. Each cooperating secondary user senses the spectrum and sends its ‘quantized’ local measurement as \( L_i \), (index of the quantization bin) to the fusion center at the cognitive base station. The fusion center makes a decision according to \( L_i \).

The global decision logic schemes in the literature can be classified into two main categories: soft decision and hard decision logic. In hard decision methods, the fusion center collects local decisions consisting of 1-bit information. In soft decision methods, the exact measurements are reported to the fusion center.

Well-known soft decision methods are EGC and maximal gain combiner. EGC uses fixed weights for measurements reported to the fusion center. All received measurements are summed coherently and compared against one global threshold. In maximal gain combiner, the received measurements are weighted with respect to their SNR values, then summed and compared with one global threshold.

If the quantization is performed with basically two bins, hard decision logic, such as or and majority logic, can be applied for global decision logic at the fusion center. Considering 0 and 1 for \( L_i \) trivially suggests hard decision logic for cooperation. However, in the case of more quantization bins, none of the hard decision logic can be used. In such a situation global decision logic must have a functional form.

We borrow the global decision logic, which we call cooperative sensing with decision vector (CSDV), from our previous work [25]. CSDV constitutes an analogy of a seesaw. The example in
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Figure 2 contains three measurements in $B_1$, one measurement in $B_2$, and five measurements in $B_3$; hence, the final decision is clearly $H_0$ even though the majority of the nodes are at the right side.

In the proposed method, the global decision depends on the threshold values and the weight vector. The reader should note that the weights are assigned to the quantization bins, not the reporting nodes.

For the given four-bin example in Figure 2, the fusion center receives the quantized measurements and counts the number of users in each quantization bin. Having three reports in $B_1$, one report in $B_2$, and five reports in $B_3$ implies

$$\sum_{B} N_B = 3 + 1 + 5 = 9$$

Having a weight vector enables us to tune the system performance and the system behavior. Clearly, $\vec{w}$ partitions $B$ (the set of all possible $\vec{B}$ vectors) into two disjoint sets as $B_0$ and $B_1$, where the final decision is $H_0$ and $H_1$, respectively. $\delta_{\vec{w}}(.)$ evaluates to 0 in $B_0$, and its corresponding terms in the probability evaluations become 0. Both $P_d$ and $P_f$ are evaluated by summing the probabilities of observing the cases in $B_1$ because $P_d$ and $P_f$ are the probabilities of deciding $H_1$ as the final decision conditioned on having primary communication and no primary communication, respectively.

$$P^c_{d,\text{CSDV}} = \sum_{B \in B_1} \left( \sum_{j=1}^k \binom{N}{N_{B_j}} \right) P^c_{H_0}(B_j)$$

$$P^c_{d,\text{CSDV}} = \sum_{B \in B_1} \left( \sum_{j=1}^k \binom{N}{N_{B_j}} \right) P^c_{H_1}(B_j)$$

3.3. Evaluation of $P_d$ and $P_f$ for CSDV

We mainly focus on the Rayleigh channel, and using CSDV method as the fusion strategy leads to the following probabilities of cooperative detection and false alarm under Rayleigh channel:

$$P^R_{\text{CSDV}} = \sum_{B \in B} \delta_{\vec{w}}(\vec{B}) \prod_{k=1}^n \left( N - \sum_{j=1}^{k-1} N_{B_j} \right) \left( P^R_{H_0}(B_k) \right)^{N_{B_k}} \left( P^R_{H_1}(B_k) \right)^{N_{B_k}}$$

$$P^R_{d,\text{CSDV}} = \sum_{B \in B} \delta_{\vec{w}}(\vec{B}) \prod_{k=1}^n \left( N - \sum_{j=1}^{k-1} N_{B_j} \right) \left( P^R_{H_0}(B_k) \right)^{N_{B_k}} \left( P^R_{H_1}(B_k) \right)^{N_{B_k}}$$
3.4. Evaluation of $P_d$ and $P_f$ for CSDV with false reports

When one of the cognitive radios (CRs), intentionally or not, reports the presence of primary user by reporting $H_1$ erroneously all the time, the or logic always decides the presence of primary user. In this case, $P_{f_{LOR}}^h = 1$ under any given channel condition, and the utilization of the white spaces becomes impossible.

Using CSDV method as the fusion strategy with $fr$ false reports leads to the following probabilities of cooperative detection and false alarm under Rayleigh channel:

$$P_{d,CSDV}^r = \sum_{B \in B^{-fr}} \left\{ \delta_{\text{Nerf}}(B + fr) \prod_{k=1}^{n} \left( N - fr - \sum_{j=1}^{k-1} N_{B_j} \right) \left( P_{H_0}^r(B_k) \right)^{N_{B_k}} \right\}$$

(19)

$$P_{f,CSDV}^d = \sum_{B \in B^{-fr}} \left\{ \delta_{\text{Nerf}}(B + fr) \prod_{k=1}^{n} \left( N - fr - \sum_{j=1}^{k-1} N_{B_j} \right) \left( P_{H_1}^r(B_k) \right)^{N_{B_k}} \right\}$$

(20)

where $N$ is the number of cooperative users, $N_{B_k}$ is the number of users having observation in bin $B_k$ without considering $fr$ nodes, $n$ is the number of quantization bins, $B^{-fr}$ represents all combinations of $N - fr$ users distributed in quantization bins, and $B + fr = B + [000fr]$ for the three-threshold (four-bin) case. We do not have any assumption on which nodes are the false reporting nodes; the formulation depends only on the number of false reporting nodes. Increasing the number of false reporting nodes degrades the system performance.

3.5. Evaluation of $P_d$ and $P_f$ for CSDV in imperfect reporting channel

When the reporting channel is imperfect, errors may occur on the reported local decisions. In case of 1-bit quantization, $L_i$ can only be 0 or 1 (corresponding to $B_1$ or $B_2$). Thus, it can be modeled as a binary symmetric channel with cross-over probability $p_e$, which is equal to bit error rate of the channel.

In the hard decision case, under $H_1$, the fusion center can receive 1 from a CR node in two situations: In the first case, CR node decides $H_1$ ($B_2$) locally and transmits it to the fusion center without any errors, which occurs with probability $P_{H_1}^h(B_2)(1 - p_e)$. The second situation occurs when CR node decides $H_0$ ($B_1$) locally and transmits it to the fusion center with error, which occurs with probability $P_{H_1}^h(B_1)p_e$. Therefore, detection probability for single node with imperfect reporting channel becomes

$$P_{H_1}^{h,e}(B_2) = P_{H_1}^h(B_2)(1 - p_e) + P_{H_1}^h(B_1)p_e.$$  

(21)

Using similar reasoning and formulation, local false alarm probability with imperfect reporting channel given as

$$P_{H_0}^{h,e}(B_2) = P_{H_0}^h(B_2)(1 - p_e) + P_{H_0}^h(B_1)p_e.$$  

(22)

In case of more quantization bins, the problem becomes complicated. A sample system model for $n$-bin quantization is depicted in Figure 3. Only the arrows originating from $B_1$ are shown to make the figure more readable.

The sum of error cases is denoted by $p_e$ whereas $p_{ij}^e$ represents the probability of receiving $B_j$ at the fusion center when $B_i$ is observed by CR node and sent over the reporting channel. Here, symbol errors occur and the symbol error rate for each symbol pair may be different according to the coding used.
Generalizing the formulation in Equations (21) and (22), we denote the probability of receiving $B_i$ at the fusion center through the imperfect reporting channel as $P_{c, H_1}(B_i)$ and $P_{c, H_0}(B_i)$ under $H_1$ and $H_0$, respectively, and evaluate

$$P_{c, H_1}(B_i) = P_{c, H_1}(B_i)(1 - p_e) + \sum_{k \neq i} P_{c, H_1}(B_k)p_{e, k}^i,$$

(23)

$$P_{c, H_0}(B_i) = P_{c, H_0}(B_i)(1 - p_e) + \sum_{k \neq i} P_{c, H_0}(B_k)p_{e, k}^i.$$

(24)

After deriving these equations, we plug $P_{c, H_0}(B_i)$ and $P_{c, H_1}(B_i)$ into $P_f$ and $P_d$ formulation in Equations (15) and (16), respectively. We evaluate $P_f$ and $P_d$ for CSDV under Rayleigh and imperfect reporting channel

$$P_{f, CSDV} = \sum_{\hat{B} \in B} \frac{\delta_{\hat{B}}}{w} \left( \prod_{k=1}^{n} \left( N - \sum_{j=1}^{k-1} N_{B_j} \right) \left( P_{R, H_0}(B_k) \right)^{N_{B_k}} \right),$$

(25)

$$P_{d, CSDV} = \sum_{\hat{B} \in B} \frac{\delta_{\hat{B}}}{w} \left( \prod_{k=1}^{n} \left( N - \sum_{j=1}^{k-1} N_{B_j} \right) \left( P_{R, H_1}(B_k) \right)^{N_{B_k}} \right),$$

(26)

where $B$ is the set of all possible $\hat{B}$'s, $N$ is the number of cooperative users, $N_{B_k}$ is the number of reports in bin $B_k$ from the fusion center’s perspective (with errors), $P_{R, H_i}(B_k)$ is the probability of having report in bin $B_k$ under $H_i$ in Rayleigh channel and $n$ is the number of quantization bins.

### 3.6. Problem formulation

Recall that the main goal of spectrum sensing is to decide between two hypotheses that the channel state is empty ($H_0$) or is actively used ($H_1$). If the conditional distributions of $O_i$ are known, the most appropriate optimality criterion for decision is Neyman–Pearson optimality that maximizes $P_{c, H_1}(B_i)$ under Rayleigh channel when there are no false reports and the reporting channel is perfect is formulated in Equation (27) when $w$ is given. When there are false reports or the reporting channel is imperfect, we portray two problem definitions and the solution methodologies. First problem aims to optimize the thresholds of the quantizer for a given $w$ and $\alpha$. Second problem aims to optimize the $\hat{w}$ by a genetic algorithm that also utilizes the first problem definition.

### 3.6.1. Threshold optimization

The corresponding optimization problem for optimizing $\lambda_1$ and $\Delta$ under Rayleigh channel when there are no false reports and the reporting channel is perfect is formulated in Equation (27) when $w$ is given. When there are false reports or the reporting channel is imperfect, we portray two problem definitions and the solution methodologies. First problem aims to optimize the thresholds of the quantizer for a given $w$ and $\alpha$. Second problem aims to optimize the $\hat{w}$ by a genetic algorithm that also utilizes the first problem definition.
imperfect, we can obtain similar optimization problem formulations by using Equations (19), (20) and (25), (26), respectively.

\[ \text{UniQCS1} : \max_{\Delta, \lambda_1} \sum_{B \in B} \delta_{\bar{w}}(B) \prod_{k=1}^{\eta} \left( \frac{N - \sum_{j=1}^{k-1} N_{B_j}}{N_{B_k}} \right) (P_{H_1}^R(B_k))^{N_{B_k}} \]

subject to \( \sum_{B \in B} \delta_{\bar{w}}(B) \prod_{k=1}^{\eta} \left( \frac{N - \sum_{j=1}^{k-1} N_{B_j}}{N_{B_k}} \right) (P_{H_0}^R(B_k))^{N_{B_k}} \leq \alpha \) \hspace{1cm} (27)

Different \((\Delta, \lambda_1)\) values change the quantization bins and the corresponding \( (P_{H_1}^R(B_k))^{N_{B_k}} \). If \( \lambda_1 = 0 \) and \( \Delta = 0 \), then we have \( P_f = 1 = P_d \). For a fixed \( \Delta \), shifting \( \lambda_1 \) and consequently the other thresholds to higher values results in decrease in \( P_f \) and \( P_d \). Similarly, for a fixed \( \lambda_1 \), shifting \( \Delta \) to higher values results in the same behavior. This behavior is the key point for solving the maximization problem and implies that the maximum \( P_{ch}^{d, CSDV} \) is achieved when \( P_{f, CSDV}^{\alpha, \lambda_1} \). We seek for \( \lambda_1^* \) and \( \Delta^* \) satisfying \( P_f = \alpha \), which gives us the maximum \( P_d \). The algorithm for finding the solution to the given problem formulation is as follows:

Require: \( \bar{w}, \alpha \)
1: \( \max PD \leftarrow 0 \)
2: \( \lambda_1^* \leftarrow 1 \)
3: \( \Delta^* \leftarrow 0 \)
4: while \( \lambda_{\text{increment}} > \text{precision} \) do
5: \( \Delta = \text{findDeltaForPFEqualsAlpha}(\lambda_1, \alpha, \bar{w}) \)
6: \( P_d = \text{evaluatePD}(\lambda_1, \Delta, \bar{w}) \)
7: if \( P_d > PD_{\text{prev}} \) then
8: \( \lambda_1_{+} = \lambda_{\text{increment}} \)
9: \( PD_{\text{prev}} = P_d \)
10: else
11: \( \lambda_1_{-} = \lambda_{\text{increment}} \)
12: Decrease \( \lambda_{\text{increment}} \)
13: \( \lambda_1_{+} = \lambda_{\text{increment}} \)
14: end if
15: if \( P_d > \max PD \) then
16: \( \max PD \leftarrow P_d \)
17: \( \lambda_1^* \leftarrow \lambda_1 \)
18: \( \Delta^* \leftarrow \Delta \)
19: end if
20: end while

where the function \( \text{findDeltaForPFEqualsAlpha}(\lambda_1, \alpha, \bar{w}) \) simply solves equation for corresponding \( \Delta \) that makes \( P_f = \alpha \) by shifting \( \Delta \) to higher values with decreasing increments. The thresholds are optimized in a search manner that finds the optimum thresholds for the proposed method in terms of \( P_d \) with given \( P_f \) and \( \bar{w} \).

3.6.2. Weight optimization. We introduce another optimization problem for optimizing \( \bar{w} \) similar to UniQCS1.

\[ \text{UniQCS2} : \max P_d \text{ subject to } P_f \leq \alpha \] \hspace{1cm} (28)
Different \( \vec{w} \) vectors change \( B_1 \), hence \( \delta_{\vec{w}}(\hat{B}) \) values, and also change \( P_d \) values evaluated optimally by UniQCS1. We optimize the \( \vec{w} \) via embedding genetic algorithm into the optimization procedure of the proposed method. Initial phase of the genetic algorithm selects initial weights. For different \( \vec{w} \) values, thresholds are optimized by UniQCS1 and rated according to \( P_d \) values and the fit \( \vec{w} \) individuals survive to next generation. We have the population size as 15 \( \vec{w} \)'s, and the crossover strategy averages the corresponding parent \( \vec{w} \)'s. In all generations, five elites that give offsprings to new population are selected according to \( P_d \) values, and one individual experiences mutation. After some iterations, \( \vec{w} \) improvement according to the parameters is insignificant and the algorithm stops.

Genetic algorithm improves \( \vec{w} \), and the thresholds are optimized according to UniQCS1 according to the system parameters such as mean SNR, number of cooperating nodes, and channel conditions. Hence, we can evaluate the optimization variables for predetermined system parameters offline. If system parameters change, new values of optimization variables are evaluated by using UniQCS.

4. RESULTS AND DISCUSSIONS

We analyze the performance of UniQCS mostly under flat fading Rayleigh channel. We evaluate the maximum \( P_d \) for a given \( P_f \) constraint. For comparisons, we include the 'no cooperation' case and EGC when necessary.

4.1. Performance metrics and system parameters

\( P_d \) is an important metric of the system because it directly affects the primary users. Low probability of detection implies high missed detection, which results in failure of the CR node in vacating the channel during primary access. Therefore, stringent conditions are imposed on the value of \( P_d \). On the other hand, having high probability of false alarm, \( P_f \), results in low utilization of white spaces. Therefore, tradeoff between these two metrics must be considered. Receiver operating characteristic (ROC) curves are used for tradeoff analysis between \( P_d \) and \( P_f \).

We consider the following system parameters in our analysis: average SNR, TW, and the number of cooperating users denoted by \( N \). UniQCS method optimizes its parameters by using the algorithms explained in Section 3.6 for UniQCS1 and UniQCS2. UniQCS2 elevates \( \vec{w} \) in a sub-optimal manner using genetic algorithm, whereas UniQCS1 finds \( (\lambda_1, \Delta) \) in an optimal manner using numerical methods.

4.2. No false reports and perfect reporting channel

We examine the performance of the proposed method under perfect reporting channel while there are no false reports.

4.2.1. Analysis of global decision logic on \( P_d \). In Figures 4 and 5, we analyze the performance of different global decision logic by using ROC curves under AWGN and Rayleigh channels, respectively. In the figures, \( P_d \) values corresponding to the given \( P_f \) constraint are plotted. UniQCS performs close to EGC far better than hard decision logic. In Rayleigh channel, or logic performs better than majority logic for 5 dB SNR because there are fewer nodes with high SNR value, favoring or logic. Starting from this point in the paper, we consider Rayleigh channel, and for the UniQCS case, we analyze the system performance considering only eight-bin quantization.

4.2.2. Effect of number of users on \( P_d \). In Figure 6, we analyze the effect of number of users on \( P_d \) for a given \( P_f \). In the figure, x-axis and y-axis correspond to the number of nodes and \( P_d \), respectively. Increase in the number of nodes results in higher \( P_d \). UniQCS and EGC exhibit similar performances where EGC is an upper bound for the case because we do not distinguish users according to their SNR values. Majority and or logic have a crossing point because SNR is exponentially distributed.
Figure 4. ROC curves of different global decision logic ($SNR = 5$ dB, $TW = 5$, $N = 10$) under AWGN channel.

Figure 5. ROC curves of different global decision logic ($SNR = 5$ dB, $TW = 5$, $N = 10$) under Rayleigh channel.

Figure 6. $N$ versus $P_d$ curves of different global decision logic under Rayleigh channel ($P_f = 0.001$, $SNR = 5$ dB, $TW = 5$).
and there are not enough CR nodes to take advantage of majority logic, which is up to 10 CR nodes. Performance gain of UniQCS is at least 50\% compared with the hard decision methods for 15 CR nodes.

4.3. Robustness against false reports

In this subsection, we examine the performance of the proposed method under perfect reporting channel while there are two false reporting nodes. Because there are two false reports, we do not include or logic in the performance figures. Or logic and 2/N logic clearly fail ($P_d = 1 = P_f$). Therefore, the utilization of white spaces becomes impossible. Hence, we include 3/N logic instead of or logic and 2/N logic for comparisons.

4.3.1. Analysis of global decision logic on $P_d$. False reports adversely affect 3/N logic more than any other logic in Figure 7. UniQCS is also affected by false reports, but UniQCS with two false reports still performs better than other methods without any false reports. Majority logic is the least affected method due to false reports. However, its performance is far below the UniQCS. For $P_f = 10^{-2}$, performance gain of UniQCS is at least 45\% compared with the other methods under false reports.

4.3.2. Effect of number of users on $P_d$. In Figure 8, similar behavior is observed as in the previous false report analysis figures. UniQCS is also affected by false reports, but UniQCS with two false reports again performs better than other methods even if they do not experience any false reports. For fewer nodes, the gain of UniQCS over other algorithms decreases. Increasing the number of cooperating nodes results in increase in robustness against false reports because the performance loss due to false reports decreases. UniQCS is robust against false reports and performs better than other methods.

4.3.3. Effect of number of false reporting nodes on $P_d$. In Figure 9, the effect of number of false reporting nodes is analyzed. The $x$-axis corresponds to the number of false reporting nodes changing from 0 to 6. It is not reasonable to consider the case of having number of false reporting nodes more than the half of the cooperating nodes. Or, 2/N, and 3/N logic do not satisfy the $P_f$ constraint after some point. Therefore, we compare UniQCS with majority logic. Clearly, increasing the number of false reporting nodes decreases $P_d$ for a target $P_f$. UniQCS performs better than majority logic and is a better option with marginal overhead bits due to quantization.

Figure 7. ROC curves of different global decision logic including false reports ($SNR = 5$ dB, $TW = 5$, $N = 10$).
Figure 8. $N$ versus $P_d$ curves of different global decision logic including false reports ($P_f = 0.01$, $SNR = 5$ dB, $TW = 5$).

Figure 9. Number of false reporting nodes versus $P_d$ curves ($P_f = 0.01$, $SNR = 5$ dB, $TW = 5$, $N = 15$).

4.4. Imperfect reporting channel

In this subsection, we examine the performance of the proposed method under imperfect reporting channel while there is no false report.

4.4.1. Analysis of global decision logic on $P_d$. In Figure 10, we compare the perfect and imperfect channel cases. Some of the curves are incomplete because the target $P_f$ is not achievable. Under imperfect channel, or logic is the most adversely affected method. Smallest $P_f$ that is achievable for or logic and 2/N logic is 0.1 and 0.005, respectively, and smaller $P_f$ values are not achievable. Performance loss due to imperfect reporting channel in UniQCS decreases as $P_f$ increases. Again, UniQCS performs better than other hard decision logic even they have perfect reporting channel.

4.4.2. Effect of SNR on $P_d$. Effect of SNR on $P_d$ is analyzed in Figure 11. As SNR increases $P_d$ increases, we observe that or logic cannot achieve $P_f = 0.01$ at all in case of imperfect reporting channel. 2/N logic is also adversely affected by $P_e = 0.01$ whereas UniQCS is robust against errors. If we consider 5 dB case, UniQCS performance gain is at least 50% compared with the other methods under imperfect reporting channel conditions. Increasing SNR results in higher $P_d$ for 2/N, but or logic cannot achieve $P_f = 0.01$ even for high SNR values.
Figure 10. ROC curves of different global decision logic including imperfect reporting channel 
\( (SNR = 5 \text{ dB}, TW = 5, N = 10) \).

Figure 11. SNR versus \( P_d \) curves of different global decision logic including imperfect reporting channel 
\( (P_f = 0.01, TW = 5, N = 10) \).

4.4.3. Effect of number of users on \( P_d \). In Figure 12, effect of number of users on \( P_d \) is analyzed while the target \( P_f \) is 0.01. Under imperfect channel, or logic is the most adversely affected method. Moreover, or logic cannot achieve \( P_f = 0.01 \) for the given parameters. As the number of CR nodes increases, 2/N logic performs worse because it is sufficient to have two nodes deciding \( H_1 \) and the probability of having two errors increases with more nodes when the average SNR evaluated after the introduction of BEP is below the SNR wall [26]. Once again, UniQCS is robust against imperfect reporting channel and performs better than other methods even if the reporting channel is perfect for other methods.

4.4.4. Effect of \( P_e \) on \( P_d \). Effect of \( P_e \) on \( P_d \) is analyzed in Figure 13. Increasing \( P_e \) decreases \( P_d \), if \( P_f = 0.01 \) is achievable. UniQCS and majority logic are robust against imperfect reporting channel. However, majority logic performance is much lower than UniQCS. The proposed method performance gain is at least 15% compared with other methods for \( P_e = 10^{-3} \). Moreover, BEP wall for UniQCS is higher than the other methods as seen from the figure.
Figure 12. $N$ versus $P_d$ curves of different global decision logic including imperfect reporting channel ($P_f = 0.01$, $SNR = 5$ dB, $TW = 5$).

Figure 13. $P_e$ versus $P_d$ curves of different global decision logic including imperfect reporting channel ($P_f = 0.01$, $SNR = 5$ dB, $TW = 5$, $N = 10$).

5. CONCLUSION

Awareness about the wireless environment is the most crucial part of the CR system because it includes sensing the environment for primary activity, finding white spaces, and vacating the channel or adjusting the communication parameters, which is a must for not disturbing the primary users or constructing the Radio environment map.

In this paper, we focus on cooperative ED and uniform quantizer improvements against low SNR conditions under imperfect reporting channel and false reports. By transmitting 2-bit or 3-bit information, proposed UniQCS method performs very close to EGC scheme, which transmits exact measurements and outperforms the hard decision methods. Our proposed method UniQCS achieves a performance close to the EGC scheme, which is an upper bound for the case because the nodes are not differentiated, at the expense of transmitting one or two extra bits compared with the hard decision methods. UniQCS is also robust against imperfect reporting channel and false reports. For probability of false alarm equal to 0.01, performance gain of UniQCS is at least 45% compared with the other methods when there are two false reporting nodes. UniQCS performance gain is at least 15% compared with other methods for probability of reporting channel error equal to 0.001.
For the future directions, we work on adaptive weights, multi-hop performance analysis for reporting the quantized measurement, and developing an optimal sensing framework.

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