

CMPE 300 ANALYSIS OF ALGORITHMS
MIDTERM

23.11.2015

1. For each of the following pairs of functions, either $f(n) \in O(g(n))$, $f(n) \in \Omega(g(n))$, or $f(n) \in \Theta(g(n))$. For each pair, determine which relationship is correct. Justify your answer.

- a) $f(n) = \log n^2$; $g(n) = \log n + 5$
- b) $f(n) = \sqrt{n}$; $g(n) = \log n^2$
- c) $f(n) = \log^2 n$; $g(n) = \log n$
- d) $f(n) = n$; $g(n) = \log^2 n$
- e) $f(n) = n \log n + n$; $g(n) = \log n$
- f) $f(n) = \log n^2$; $g(n) = (\log n)^2$
- g) $f(n) = 10$; $g(n) = \log 10$
- h) $f(n) = 2^n$; $g(n) = 10n^2$
- i) $f(n) = 2^n$; $g(n) = n \log n$
- j) $f(n) = 2^n$; $g(n) = 3^n$
- k) $f(n) = 2^n$; $g(n) = n^n$

2.

- a) Give the definition of the Master Theorem.
- b) Solve the following recurrence relation using the Master Theorem.
 $T(n) = 3 T(n/5) + 8 n^2$, $T(1)=c$
- c) Solve the following recurrence relation using the Master Theorem.
 $T(n) = 2 T(n/2) + n$, $T(1)=c$
- d) Find the exact closed-form solution for the following recurrence relation. Assume that n is a power of 2. (Do not use the Master Theorem.)
 $T(n) = 2 T(n/2) + n$, $T(2)=2$

3. Find the worst-case complexity of Binary Search for any data size n . You must find the exact complexity. (If interpolation is necessary, do not use the interpolation theorem; show it step by step.)

4. Counting Sort algorithm is as follows: We have a list $L[1:n]$ of size n and we assume that the list elements are integers in the range from 1 to k . In the first pass, the algorithm counts the number of elements with each input value and stores these counts. In the second pass, the algorithm uses these counts to place the elements in their correct positions.

Example: Suppose that $L=[5, 8, 8, 2, 4, 5, 4, 8, 5, 5]$, $n=10$, $k=8$.

1st pass: The algorithm calculates that “there is 1 element with value 2”, “there are 2 elements with value 4”, “there are 4 elements with value 5”, “there are 3 elements with value 8”, and there are 0 elements with other values between 1 and 8.

2nd pass: The algorithm puts the sorted values in an output list. One position in the output list will be reserved for value 2; beginning from position 2, two positions will be reserved for value 4; beginning from position 4, four positions will be reserved for value 5; beginning from position 8, three positions will be reserved for value 8. Then the values are inserted.

Write this algorithm in pseudocode. Analyze its complexity. In which cases this algorithm is efficient?

(continued on next page)

Notes:

- Where pseudocode is required, the syntax of the pseudocode must be strictly followed. No points will be given if the syntax is not followed or any other language (e.g. C) is used.
 - Questions 1-4 : 25 points
 - Time: 1:50 hours
 - Close notes and books
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Kim bilir ne oldu ne bitti gece uyurken
Uyandım ki tabiat o de il ba kasıdır
Nerden esiyor bu erbet gibi hava nerden
Çiçek açmı a aç hangi kızın rüyasıdır.

Güzelli ine imrendi imiz dünyasında
Çocu un saltanatını gör arabasında
Ne var ki u geçen yosmanın kırıtmasında
Gencin ihtirası ya lının hâtırasıdır.

Gel böyle havada kendini emniyette bil
Bir bakı bir gülü le çarpılmak i ten de il
Çekti im a rı kalpten bermutad di ten de il
Bildim bileli bahar ba ımın belâsıdır.