

CMPE 300 - Analysis of Algorithms
Fall 2015
Assignment 1

Due: November 11, 17:00

Question 1

- a) Write an algorithm in pseudocode for computing the k 'th power of a square matrix of dimension $n \times n$. The runtime of your algorithm should be $o(kn^3)$. Perform an exact analysis and express the time complexity in big O notation.
- b) Let A and B be integer square matrices of dimension $(n + 2) \times (n + 2)$ which have the following form:

$$\begin{bmatrix} 1 & \mathbf{a} & c \\ 0 & I_n & \mathbf{b} \\ 0 & 0 & 1 \end{bmatrix}$$

where \mathbf{a} is an n dimensional row vector, \mathbf{b} is an n dimensional column vector, $c \in \mathbb{Z}$, and I_n is the identity matrix of dimension n . An example for dimension 7 can be given as follows:

$$\begin{bmatrix} 1 & 3 & 4 & 0 & 1 & -2 & 5 \\ 0 & 1 & 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Describe a method for multiplying A and B which requires $O(n)$ time.

Question 2

Consider the given function $f(n)$ and determine whether the following cases are true or false. Justify your answers formally.

$$f(n) = n^2 \log n + n^3 \sum_{i=1}^n \frac{1}{i} + n^3 \sum_{i=0}^n \frac{1}{2^i}$$

- a) $f(n) \in O(n^4)$
- b) $f(n) \in \theta(n^4)$
- c) $f(n) \in \Omega(n^3 \log n)$
- d) $f(n) \in o(n^4 \log n)$