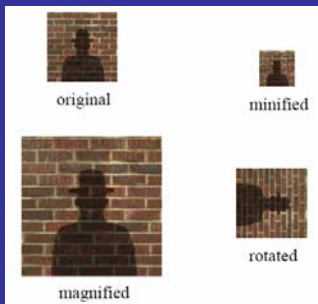


## Image Warping

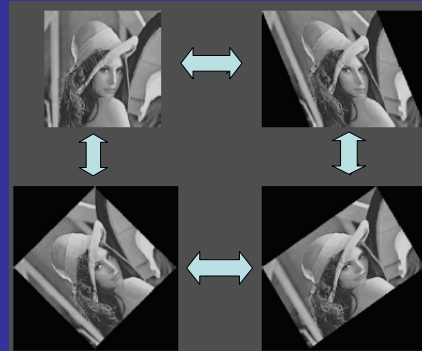
## Image Warping

- Warping: The mapping from signal  $F(x)$  to signal  $G(\zeta)$  by some transformation  $\zeta=m(x)$  satisfying  $G(m(x)) = F(x)$ .
- Image warping:
  - $x=(x,y), \zeta=(\zeta, \eta), x, y, \zeta, \eta \in I$
  - $F(x), G(\zeta) \in [0, 255]$
  - $m()$ :
    - Linear operation:  $\zeta=Mx$
    - Piecewise linear operations:  $\zeta=M(x)x$
    - RBF approximation:  $\zeta = \sum_i a_i g_i(\|x-x_i\|)$  [4]

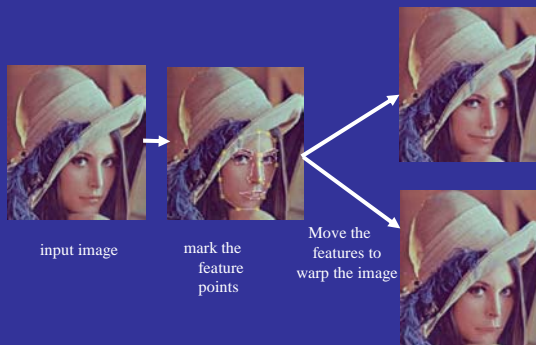
## Warping Example



## Warping Example



## Warping Example



## Image warping



- Two warping procedures:
  - For each  $x, \zeta=Mx, F(x) = G(\zeta)$
  - For each  $\zeta, x=M^{-1}\zeta, F(x) = G(\zeta)$ . *This one is bad, because  $x$  can be out of the target image region.*

## Affine Warping – definition

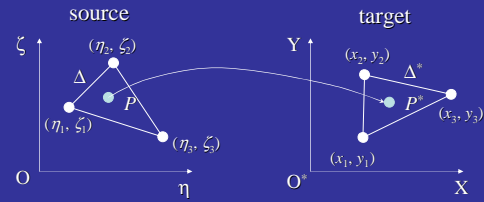
- Assume the mapping from  $(x, y)$  to  $(\zeta, \eta)$  is linear:

$$\begin{aligned}\zeta &= a_1x + a_2y + a_3 \\ \eta &= b_1x + b_2y + b_3\end{aligned}$$

$$\begin{bmatrix} \zeta \\ \eta \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

6 Unknowns  $(a, b)$  need to be solved by 6 equations, e.g. need 3 pair of correspondences

## Affine Warping



Correspondences:  $(\eta_1, \zeta_1) \rightarrow (x_1, y_1)$ ,  
 $(\eta_2, \zeta_2) \rightarrow (x_2, y_2)$ ,  
 $(\eta_3, \zeta_3) \rightarrow (x_3, y_3)$ ,  
 $\Delta \rightarrow \Delta^*$

## Affine Warping – better representation

For any  $(\zeta, \eta)$  in the triangle, we have

$$\begin{aligned}\zeta &= \lambda_1 \zeta_1 + \lambda_2 \zeta_2 + \lambda_3 \zeta_3 \\ \eta &= \lambda_1 \eta_1 + \lambda_2 \eta_2 + \lambda_3 \eta_3 \\ 1 &= \lambda_1 + \lambda_2 + \lambda_3 \quad (\lambda_i \geq 0, i = 1, 2, 3)\end{aligned} \quad (2)$$

or

$$\begin{bmatrix} \zeta \\ \eta \\ 1 \end{bmatrix} = \lambda_1 \begin{bmatrix} \zeta_1 \\ \eta_1 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} \zeta_2 \\ \eta_2 \\ 1 \end{bmatrix} + \lambda_3 \begin{bmatrix} \zeta_3 \\ \eta_3 \\ 1 \end{bmatrix} = \begin{bmatrix} \zeta_1 & \zeta_2 & \zeta_3 \\ \eta_1 & \eta_2 & \eta_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

## Affine Warping – better representation

- This representation is invariant to affine transformation, thus  $P^*$ 's corresponding point  $P^*(x, y)$  can be represented as

$$\begin{aligned}x &= \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 \\ y &= \lambda_1 y_1 + \lambda_2 y_2 + \lambda_3 y_3\end{aligned}$$

with the same  $\lambda_i$  (3)

$$\begin{bmatrix} \zeta \\ \eta \\ 1 \end{bmatrix} = \begin{bmatrix} \zeta_1 & \zeta_2 & \zeta_3 \\ \eta_1 & \eta_2 & \eta_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = M \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = M \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

## Affine Warping

Affine Warping in Matrix format

Let  $\mathbf{V} = (\zeta, \eta, 1)^T$ ;  $\mathbf{V}' = (x, y, 1)^T$ ;  
 $\mathbf{A} = [\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3]$ ;  $\mathbf{B} = [\mathbf{V}'_1, \mathbf{V}'_2, \mathbf{V}'_3]$ ;  $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)^T$ ;

We then have

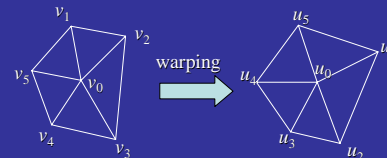
$$\mathbf{A}\boldsymbol{\lambda} = \mathbf{V}; \quad \mathbf{B}\boldsymbol{\lambda} = \mathbf{V}';$$

Thus the affine warping is:

$$\mathbf{V} = \mathbf{M}\mathbf{V}' = \mathbf{A}\mathbf{B}^{-1}\mathbf{V}';$$

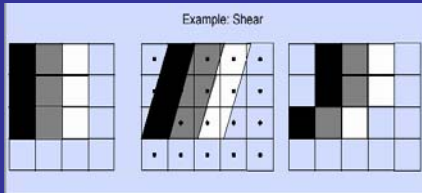
## Affine Warping

Piecewise warping: each triangle defines a warping.

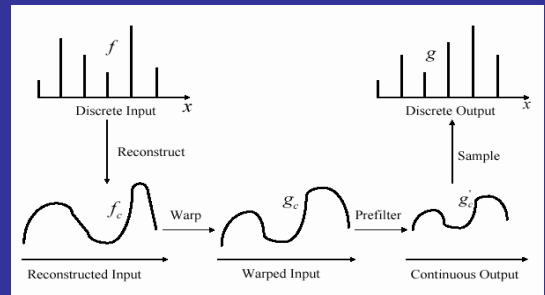


Problem: possible distortion along the boundaries of the triangles

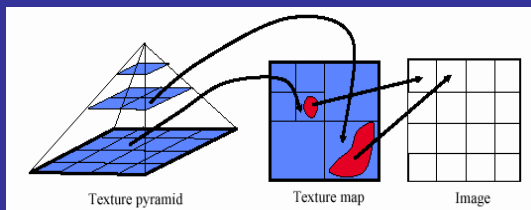
## Resampling is a problem



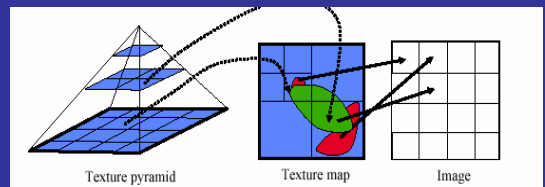
## Ideal warping



## Image pyramids



## Trilinear pyramid interpolation



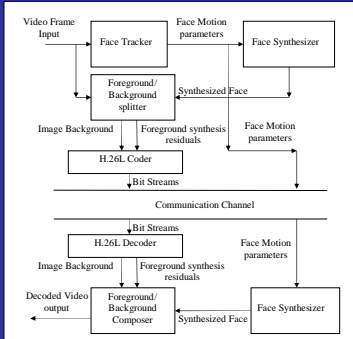
## Reference

- [1] F. I. Parke, "Parameterized models for facial animation," *IEEE Computer Graphics and Applications*, vol. 2, no. 9, pp.61-68, Nov. 1982.
- [2] Y. Lee, D. Terzopoulos, and K. Waters, "Realistic modeling for facial animation," in *Proc. SIGGRAPH 95*, pp. 55-62, 1995.
- [3] Kalra, A. Mangili, N. M. Thalmann, and D. Thalmann, "Simulation of facial muscle actions based on rational free form deformations," in *Proc. EUROGRAPHICS'92*, pp. 59-69, Sep. 1992.
- [4] N. Arad, N. Dyn, *et al.*, "Image Warping by Radial Basis Functions: Application to Facial Expressions," *Graphical Models and Image Processing*, vol. 56, no. 2, March 1994, pp. 161-172.

Application: Model based Video Coding based on visual tracking

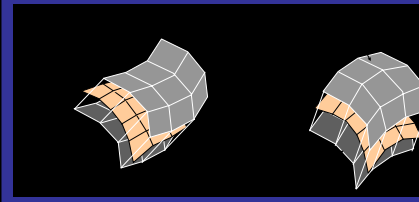
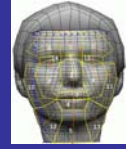
## Basic Idea

- System overview



## Basic Idea

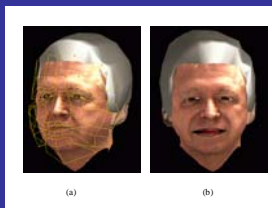
- Face Tracker
  - Piece-wise Bezier Volume Deformation Face Model
  - Purpose: To design FACS motion units



$$\hat{R}(\mathbf{V}_0 + L\hat{P}) + \hat{T}$$

## Basic Idea

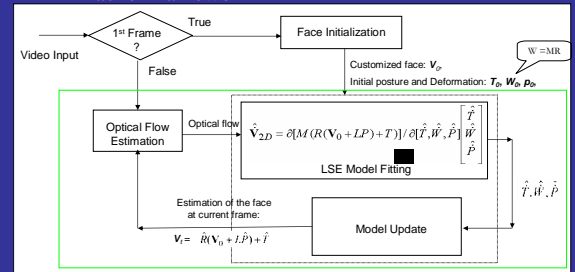
- Face Tracker
  - A example of PBVD Face Model



(a) Bezier controlling mesh (b) An expression "smile".

## Basic Idea

- Face Tracker
  - Tracker framework



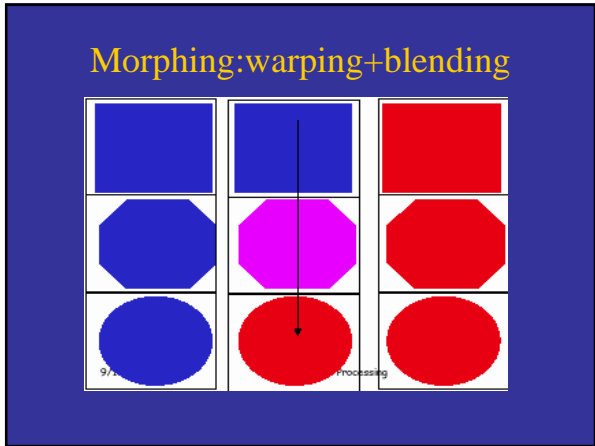
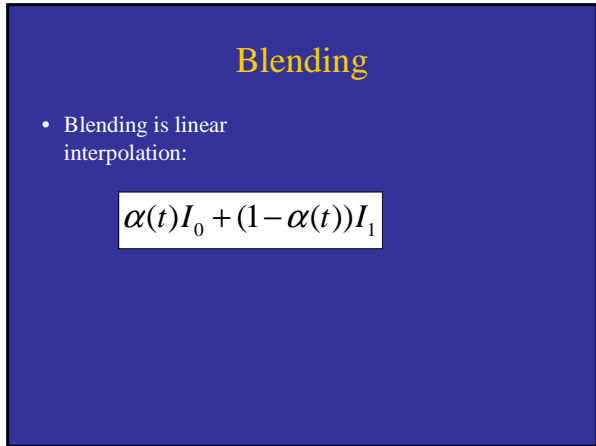
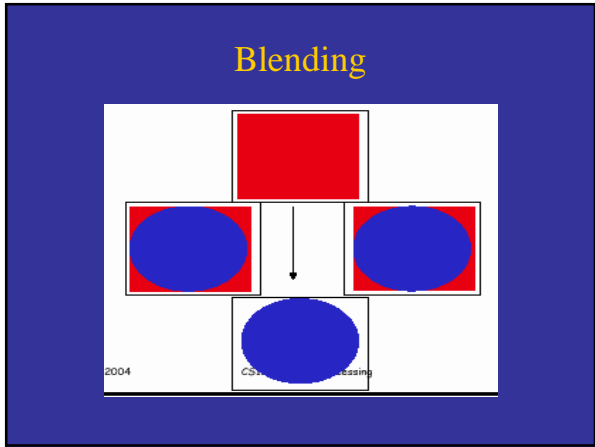
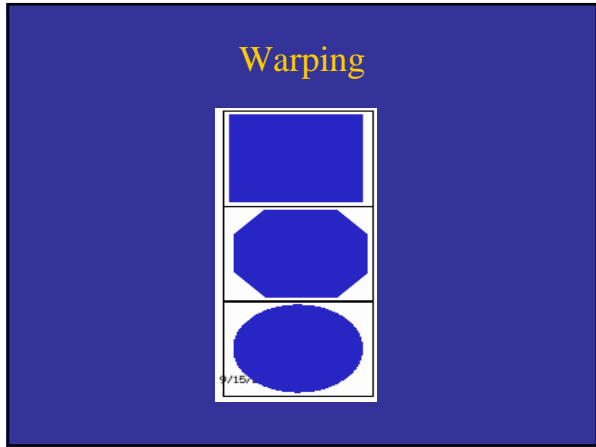
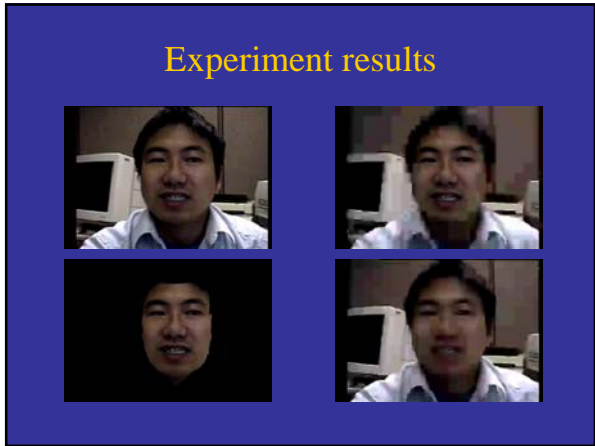
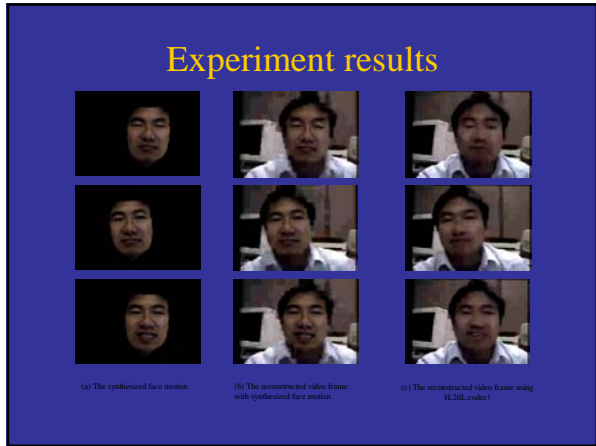
## Basic Idea

- Embedding synthesized face in H.26L Codec
  - Let H.26L take care the background efficiently.
  - The foreground reconstruction residuals coded with Intra\_16X16 mode.

## Experiment results

- Tracker: robust tracking at 25fps in rigid tracking mode and 14 fps in non-rigid mode on a PC with CPU of 2GHz.
- H.26L(JM42): IMTC ftp website: <http://ftp.imtc-files.org>.

	Coding with facial tracker	H.26L
Video Input	352X240	
Bit rate	18-19kbits/second	
PSNR around facial area	29.28	27.35
Coding speeding	1.4 sec/frame	5 sec/frame



## Morphing example

