I. CmpE 360 Homework III Answers

A. 2.12

Since each lower triangular entry in L enters into exactly one multiplication (and one addition), the number of multiplications (and additions) is simply the number of lower triangular entries, which is

$$\sum_{k=1}^{n} k = n(n+1)/2 = n^2/2 + n/2 \tag{1}$$

whose dominant term is $n^2/2$.

B. 2.13

First solve the lower triangular system $L_1 x = b$ for x by forward-substitution, then solve the lower triangular system $L_2 y = c - Bx$ for y by forward-substitution.

C. 2.17

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} = LU \tag{2}$$

D. 2.22

At step k of LU factorization by Gaussian elimination, about $(n-k+1)^2$ multiplications (and a simliar number of additions) are required. Thus, the total number of multiplications required is given by

$$\sum_{k=1}^{n} (n-k+1)^2 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \tag{3}$$

so the dominant term is $n^3/3$ multiplications (and a similar number of additions).