

**CmpE 343**  
**Fall 2008**  
**Problem Session#1**

**Question1:** You throw  $6n$  dice at random. Find the probability that each number appears exactly  $n$  times.

**Question2:** Four mice are chosen (without replacement) from a litter containing two white mice. The probability that both white mice are chosen is twice the probability that neither is chosen. How many mice are there in the litter?

**Question3:** You have  $n$  urns, the  $r$ th of which contains  $r - 1$  red balls and  $n - r$  blue balls,  $r = 1, \dots, n$ .

- (a) You pick an urn at random and remove two balls from it without replacement. Find the probability that the two balls are of different colors.
- (b) Find the same probability when you put back a removed ball.

**Question4:** Parliament contains a proportion  $p$  of Party  $A$  members, incapable of changing their opinions about anything, and  $1 - p$  of Party  $B$  members changing their minds at random, with probability  $r$ , between subsequent votes on the same issue. A randomly chosen parliamentarian is noticed to have voted twice in succession in the same way. Find the probability that (s)he will vote in the same way next time.

**Question5:** Two darts players throw alternately at a board and the first to score a bull wins. On each of their throws player  $A$  has probability  $p_A$  and player  $B$   $p_B$  of success; the results of different throws are independent. If  $A$  starts, calculate the probability that (s)he wins.

**Question6:** The joint probability density function of  $X$  and  $Y$  is given by

$$f(x, y) = \frac{6}{7} \left( x^2 + \frac{xy}{2} \right) \quad 0 < x < 1, 0 < y < 2.$$

- (a) Verify that this is indeed a joint density function.
- (b) Compute the density function of  $X$ .
- (c) Find  $\Pr(X > Y)$ .
- (d) Find  $\Pr\left(Y > \frac{1}{2} \mid X < \frac{1}{2}\right)$ .